Grid Optimization Competition
Challenge 3 Problem Formulation

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1 Introduction

This report contains the problem formulation for the Grid Optimization (GO) Competition Challenge 3. The Grid Optimization Competition is run by a team of researchers from a number of organizations, including the sponsor Advanced Research Projects Agency - Energy (ARPA-E), lead organization Pacific Northwest National Laboratory (PNNL), and technical contributors from Los Alamos National Laboratory (LANL), National Renewable Energy Laboratory (NREL), Texas A&M University (TAMU), Georgia Institute of Technology (GT), University of Wisconsin (UW), and others. The GO Competition poses challenge problems in the field of power grid management, invites entrants to develop solvers for these problems, invokes the solvers on a set of problem instances using common hardware, ranks the solvers according to their performance, and awards prizes according to the rankings. The overall goal of the GO Competition is to spur innovative research on high impact and computationally challenging problems in power grid management from initial development through commercial deployment. Complete information about the GO Competition can be found online at [2]. The webpage covers previous Challenges, rules, timeline, registration information, data formats, scoring methods, computational platform information, information on supported solvers and languages, sponsor information, frequently asked questions, administrator contact information, publicly available problem instances, computer code for reading and evaluating problem and solution data, a sandbox for testing solvers, and a solver submission interface, results, and publications.

The GO Competition has run three challenges. This report covers the Challenge 3 model formulation. In Section 2, we give a high level description of the Challenge 3 problem. In Section 3, we provide a reference to the symbols used in the formulation. In Section 4, we formulate the optimization model, specifying the variables, constraints, and objective. In Section 5, we describe the input and output data as well as further data constructed from the input data. In Section 6, we list properties that the input data should be assumed to satisfy. In Section 7, we describe the requirements a solver needs to meet. In Section 8, we describe the software for evaluation of solutions. In Section 9, we provide a log of changes to this document.

2 Problem Description

In this section, we describe the GO Competition Challenge 3 problem at a high level and explain our motivation for including certain features.

The GO Competition Challenge 3 problem is an optimization problem for short term planning of an electrical power system with special model features intended to reflect the future needs of grid planners in a rapidly changing electric power sector. We consider the power system at the transmission level, with suppliers such as generators, and consumers such as local utilities and large industrial facilities, all connected by a network of transmission lines. In the overall application of short term planning, we are focused on decisions that might be made several minutes to several days ahead of time, mainly which generators to start up or shut down how much power to dispatch from each one, but we also consider decisions such as load dispatch and line switching that might become more salient in a future power grid. By casting the problem as an optimization problem, we are looking for a plan that is the best among all possible plans according to a defined objective such as the total cost of generation. More generally, we consider dispatchable load, so we also include the total value of load served, and therefore we optimize the total market surplus.

This problem would traditionally be called an alternating current (AC) security constrained (SC) unit commitment (UC) problem. UC means we are considering the commitment of generating units through discrete decisions on startup and shutdown as well as their dispatch through continuous decisions on real power output. AC modeling covers the dispatch, flow, and balance of reactive power in addition to real power, along with voltage magnitude. SCs are constraints that ensure the planned operating point stays within safe operating limits in normal conditions and credible outage contingencies.

We consider a UC problem in a typical multi-period context, with discrete time intervals covering a planning horizon of a few hours up to ten or more days. We specifically identify three applications based on the time horizon. A 24 to 48 hour horizon with 15 minute to 2 hour time intervals is used for the day ahead planning context common in wholesale electricity markets. A 4 to 8 hour horizon with 15 minute to 1 hour time intervals is used for the near real time look ahead context where, for example, additional resources are brought online to handle revised forecasts of wind and solar output. A 5 to 10 day horizon with 1 hour to 6 hour intervals is used for a week ahead planning process that might be used especially to prepare for severe weather events.

In UC, the discrete decisions of generator startup and shutdown are modeled with binary variables, so UC is a problem of mixed integer programming (MIP). As a MIP problem UC is NP hard. This implies that solution algorithms can take too long to reach a solution with a proof of optimality within a prescribed tolerance. The practical performance of modern MIP solvers on UC is typically much better than the theoretical worst case, but there is no guarantee of solver performance, and long run times can and sometimes do happen. This poses an ongoing challenge for UC applications.

AC means we consider not only the real power output of generators and the physics of real power balance and flow but also reactive power and voltage constraints. AC modeling is typically not used in short term planning in current practice, but there is reason to believe it can permit more efficient use of generation and transmission resources in the current grid and that it may become more important as the power grid evolves.

In general, after commitment and dispatch decisions are made, the state of the power grid is realized and follows physical laws of AC power flow and balance. These physical laws are formulated as nonlinear equations in the variables of device-level real and reactive power and bus-level voltage magnitude and angle. As an optimization problem, the optimal dispatch of generators and loads (even fixed loads) subject to AC physics is a nonconvex nonlinear programming (NLP) problem. In terms of computational difficulty this problem is NP-hard.

Without the physical laws of AC power flow and balance, it is not possible to model the bus voltage magnitudes and reactive power capabilities and requirements of devices. In particular the DC power flow and balance model typically used in UC and economic dispatch models in power grid operational planning at the time scales we consider here cannot model voltage and reactive power. In practice, when a dispatch solution obtained from a DC model is implemented, the resulting bus voltages may violate engineering limits, and real time control mechanisms keep the voltages in acceptable ranges. These control mechanisms depend on the reactive power and other capabilities and requirements of devices such as generators, shunts, and loads. If these capabilities are used to their limits, then the real time controls will no longer be able to maintain voltages within the desired ranges, and damage to grid equipment and cascading grid failure can result. If reactive power and voltage were considered at the dispatch planning stage by incorporating AC power flow and balance in the UC and economic dispatch models, then the likelihood of this failure mode could be decreased. In particular, additional generators might be committed in order to make their reactive power capability available. Furthermore, with reactive power capability characteristics (for example D-curves), the real power dispatch might be modified in order to enable any given device to provide more reactive power.

In the past, it has been unnecessary to consider voltage and reactive power at the dispatch planning stage because it has been possible to ensure sufficient reactive power capability mostly by relying on static reactive power infrastructure such as shunts but also by applying special constraints to ensure certain generators are committed in thoroughly studied and well understood situations of voltage stress and by generally applying conservative flow limits to transmission lines. We believe that changes underway in the electric power industry will pose a challenge to this method of handling voltage and reactive power after planning the real power dispatch. Specifically, the geographical and temporal variability of wind and solar power and the load flexibility that might be needed to manage this variability will lead to a much greater diversity of dispatch conditions and operating points, so that fixed infrastructure and deep offline study of all credible voltage stress scenarios will no longer be practical. Furthermore, even in the current power grid, AC modeling will permit the use of less conservative flow limits on transmission lines allowing in turn more efficient dispatch and commitment decisions.

Therefore, we include a full AC power flow and balance model at each time step of the UC model. In terms of theoretical computational complexity, UC and ACOPF are already hard problems on their own, and combining them does not change this theoretical complexity. However, UC and ACOPF on their own can each be modeled and solved with solver software at a high level of commercial and academic maturity. Commercial solvers for MIP have benefited from decades of intense development spurred by high value applications throughout modern society, and mature and robust NLP solvers are also available. Practical instances of UC and ACOPF can be solved in this way with reasonable performance. Solvers for the combined MINLP problem are at an early stage of development, so that modeling a combined UC-AC problem in a straightforward fashion and passing the model to a general purpose MINLP solver is not successful on any but the smallest problem instances. With this increased practical difficulty, in addition to the theoretical complexity, this problem fits the ARPA-E model of spurring research on the hardest problems.

SC refers to a wide variety of constraints ensuring not only that the dispatch plan is consistent with physical laws of electricity but also that the resulting system state remains within safe operating limits established by engineering practice. In the UC context, SCs can be viewed narrowly as the constraints ensuring that power flows on transmission lines do not exceed predetermined limits either in the base case when the network equipment is all in operation or in any of a set of credible contingencies each defined by the unplanned outage of one or more pieces of transmission equipment.

More generally, the concept of security to credible contingencies also requires that we consider the unplanned outage of generation equipment, and our model handles this in the traditional way by requiring reserves of generation capacity that could be called up in case of a generator outage to replace the power that was being provided by that outaged generator. Furthermore, the concept of safe operation requires that we consider voltage limits with AC modeling. Finally, for simplicity, our post-contingency model for line outage contingencies is a real power only model and thus cannot resolve post-contingency bus voltages, so we introduce reactive power reserve requirements to ensure that safe voltages can be maintained in a contingency.

The future power grid is expected to be more reliant on wind and solar, which are attractive for their near zero incremental energy cost and greenhouse gas emissions but that also have high variability in available power output. Therefore, it is anticipated that dispatchable load will be critical to maintaining the balance of energy supply and demand at all times. Furthermore, some of the types of loads that are likely to have the most potential for dispatchability are large industrial plants, such as metallurgical smelters, cement manufacturers, chemical processors and refiners, and even carbon capture and sequestration equipment. Such loads may have significant operational complexity analogous to the startup, shutdown, and minimum uptime and downtime constraints of generators that are familiar in UC.

We therefore model dispatchable loads with all the same features as generators. From the standpoint of computational complexity and practical algorithmic performance, the main dimensions determining the computational difficulty of a UC problem are the number of generators and the number of time intervals. Therefore, this model feature reflecting a future user need might transform a fairly difficult but reasonable UC problem of 1000 generators and 24 or 48 time intervals into an enormously difficult problem of 5000 generator-like producing and consuming devices.

A further implication of the increasing reliance on wind and solar is that the topology of the power grid may need to be changed frequently in response to weather conditions in order to take best advantage of the available wind and solar energy. The greater diversity of dispatch conditions caused by the variability of wind and solar means that some lines should be switched (i.e. either connected or disconnected) in order to permit more power flow overall. We therefore include a decision variable to open or close each line in the network at each time step. In current practice, lines are typically not opened or closed due to day ahead or week ahead planning, and intense offline study is required to ensure that such topology switching actions can be performed without adversely affecting the dynamic state of the grid. We believe that much of the line switching analysis could be brought into the daily and weekly planning processes. Our competition takes a step towards doing that by including topology switching in the formulation to investigate the value that topology optimization could provide.

3 Nomenclature

This section gives a complete reference to the symbols used in the model formulation, in tabular form.

Units of measurement are given in Table 1, symbol main letters in Table 2, symbol superscripts in Table 3, indices and index sets in Table 4, subsets in Table 5, special set elements in Table 6, real-valued parameters in Table 7, and variables in Table 8.

Table 1: Units of measurement

Symbol	Description
binary	Binary quantities, i.e. those taking values in $\{0, 1\}$.
dimensionless	Dimensionless real number quantities.
h	hour. Time quantities are expressed in h.
integer	Integer quantities, i.e. those taking values in $\{\dots, -2, -1, 0, 1, 2, \dots\}$.
pu	per unit. Certain physical quantities, including voltage magnitude, real power, reactive power, apparent power, resistance, reactance, conductance, and susceptance, are expressed in a per unit convention with a specified or implied base value, indicated by a unit of pu.
rad	radian. Voltage angles and differences are expressed in rad.
$	US dollar. Cost, value, penalty, and objective values are expressed in $.

Table 2: Main letters

Symbol	Description
F	Set of downtime-dependent startup states
I	Set of buses
J	Set of devices
K	Set of contingencies
M	Set of cost blocks, i.e. constant marginal cost blocks of convex piecewise linear cost functions
N	Set of reserve zones
T	Set of time steps
W	Set of miscellaneous constraints
a	Point in time
b	Susceptance
c	Cost coefficient
d	Duration of time
e	Energy, e.g. stored or total produced
f	Downtime-dependent startup state
g	Conductance

Table 2 continued

Symbol	Description
i	Bus
j	Device
k	Contingency
m	Cost or value block
n	Reserve zone
p	Real power
q	Reactive power
r	Resistance
s	Apparent power
t	Index of time points and intervals
u	Integer quantities
v	Voltage magnitude
w	Index on miscellaneous constraints
x	Reactance
z	Cost, value, penalty, or objective
α	Slack distribution factor
β	Sensitivity of reactive power to real power
ϵ	Tolerance or threshold for a number to be considered nonzero
θ	Voltage angle
σ	Reserve requirement coefficient
τ	Winding ratio of a transformer
ϕ	Phase difference of a transformer

Table 3: Superscripts

Symbol	Description
ac	Alternating current (AC) branch
beta	Relating to β, the linear coefficient in linear constraints linking real and reactive power
br	Branch
ctg	Contingency
ch	Charging susceptance in an AC branch
constr	Constraint
cs	Consuming

Table 3 continued

Symbol	Description
dc	Direct current (DC) line
dn	Down, as in ramping down or reserve down
e	Energy
en	Energy, convex or concave energy cost or value function
end	End time of a time interval or last interval of the time horizon
fpd	Fixed phase difference transformer
fr	From, side of a branch
fwr	Fixed winding ratio transformer
int	integer
ln	AC line
max	Maximum
min	Minimum
mr	Must run
ms	Market surplus
nsc	Non-synchronized reserve
off	Offline
on	Online, dispatchable
out	Out of service
p	Real power
pqe	Equality constraints linking real and reactive power
pqmax	Inequality constraints modeling upper bounds on reactive power depending on real power
pqmin	Inequality constraints modeling lower bounds on reactive power depending on real power
pr	Producing
p0	Indicates a value taken by a quantity depending on real power when real power is 0
q	Reactive power
qrd	Reactive power reserve down
qru	Reactive power reserve up
rd	Ramp down
req	Requirement
rgd	Regulation down
rgu	Regulation up
rrd	Ramping reserve reserve down
rru	Ramping reserve reserve up

Table 3 continued

Symbol	Description
ru	Ramp up
s	Apparent power
scr	Synchronized reserve
sd	Shutdown, transition from online to offline or from closed to open
sdpc	Shutdown power curve
sh	Shunt
sr	Series element in AC branch model
start	Start time of a time interval or first interval of the time horizon
su	Startup, transition from offline to online or from open to closed
sus	Downtime-dependent startup state
susd	Startup/shutdown
supc	Startup power curve
time	Time
to	To, side of a branch
tr	Transition between modes
unit	Unit, as in minimum time unit
up	Up, as in ramping up or reserve up
v	Voltage (magnitude)
vpd	Variable phase difference transformer
vwr	Variable winding ratio transformer
xf	Transformer
0	Initial value, referring some period immediately prior to the model horizon
+	Slack variable on an inequality constraints, or largest slack among a set of inequalities

Table 4: Index sets

Symbol	Description
$f \in F$	Downtime-dependent startup states.
$i \in I$	Buses.
$j \in J$	Bus-connected grid devices, e.g. loads, generators, lines.
$k \in K$	Contingencies.
$m \in M$	Offer or bid blocks of piecewise linear convex cost or value functions.
$n \in N$	Reserve zones.

Table 4 continued

Symbol	Description
$t \in T$	Time intervals.
$w \in W$	Miscellaneous constraints.

Table 5: Subsets

Symbol	Description
$F_j \subset F$	Downtime-dependent startup states of device j
$I_n \subset I$	Buses contained in reserve zone n
$J_i \subset J$	Devices connected to bus i
$J_t \subset J$	Devices in service in time interval t
$J_k \subset J$	Devices that are in service in contingency k
$J^{\mathrm{ac}} \subset J$	AC branches
$J_k^{\mathrm{ac}} \subset J$	AC branches that are in service in contingency k
$J^{\mathrm{br}} \subset J$	Branches, i.e. devices connecting to two buses
$J_k^{\mathrm{br}} \subset J$	Branches that are in service in contingency k
$J^{\mathrm{cs}} \subset J$	Consuming devices (e.g. loads)
$J^{\mathrm{dc}} \subset J$	DC lines
$J_k^{\mathrm{dc}} \subset J$	DC lines in service in contingency k
$J^{\mathrm{fpd}} \subset J$	Transformers having fixed phase difference
$J^{\mathrm{fwr}} \subset J$	Transformers having fixed winding ratio
$J_k^{\mathrm{out}} \subset J$	Devices that are outaged by contingency k
$J^{\mathrm{pqe}} \subset J$	Producing or consuming devices having linear equality constraints linking real and reactive power
$J^{\mathrm{pqmax}} \subset J$	Producing or consuming devices having linear inequality constraints modeling upper bounds on reactive power depending on real power
$J^{\mathrm{pqmin}} \subset J$	Producing or consuming devices having linear inequality constraints modeling lower bounds on reactive power depending on real power
$J^{\mathrm{pr}} \subset J$	Producing devices (e.g. generators)
$J_n^{\mathrm{pr}} \subset J$	Producing devices contained in reserve zone n
$J^{\mathrm{ln}} \subset J$	AC lines
$J^{\mathrm{sh}} \subset J$	Shunts
$J_i^{\mathrm{to}} \subset J$	Branches with to bus equal to bus i
$J^{\mathrm{vpd}} \subset J$	Transformers having variable phase difference

Table 5 continued

Symbol	Description
$J^{\mathrm{vwr}} \subset J$	Transformers having variable winding ratio
$J^{\mathrm{xf}} \subset J$	Transformers
$J^{\mathrm{pr,cs}} \subset J$	Devices that are either producing devices or consuming devices
$J^{\mathrm{pr,cs,ac}} \subset J$	Devices that are either producing devices, consuming devices, or AC branch devices
$J_i^{\mathrm{fr}} \subset J$	Branches with from bus equal to i.
$J_i^{\mathrm{to}} \subset J$	Branches with to bus equal to i.
$J_i^{\mathrm{br}} \subset J$	Branches incident to bus i.
$J_i^{\mathrm{pr}} \subset J$	Producing devices at bus i.
$J_i^{\mathrm{cs}} \subset J$	Consuming devices at bus i.
$J_i^{\mathrm{sh}} \subset J$	Shunts at bus i.
$J_i \subset J$	Devices connected to bus i.
$J_n^{\mathrm{pr}} \subset J$	Producing devices in reserve zone n.
$J_n^{\mathrm{cs}} \subset J$	Consuming devices in reserve zone n.
$M_{jt} \subset M$	Energy cost or value function offer or bid blocks for producing or consuming device j in interval t
$N^{\mathrm{p}} \subset N$	Reserve zones for products associated with real power
$N^{\mathrm{q}} \subset N$	Reserve zones for products associated with reactive power
$N_i \subset N$	Reserve zones containing bus i
$T_j^{\mathrm{out}} \subset T$	Time intervals in which device j is out of service
$T_j^{\mathrm{mr}} \subset T$	Time intervals in which device j is must-run
$T_{jt}^{\mathrm{dn,min}} \subset T$	Time indices $t' < t$ such that if device j shuts down in interval t' then the minimum downtime precludes starting up in interval t
$T_{jt}^{\mathrm{up,min}} \subset T$	Time indices $t' < t$ such that if device j starts up in interval t' then the minimum uptime precludes shutting down in interval t
$T_{jt}^{\mathrm{supc}} \subset T$	Time indices $t' > t$ such that if device j starts up in interval t' then the startup power curve has nonzero power in interval t
$T_{jt}^{\mathrm{sdpc}} \subset T$	Time indices $t' \leq t$ such that if device j shuts down in interval t' then the shutdown power curve has nonzero power in interval t
$T_{jf}^{\mathrm{sus}} \subset T$	Time indices t such that a constraint is needed to ensure that if device j is starting up in state f in time interval t then it was online in some sufficiently recent prior interval
$T_{jft}^{\mathrm{sus}} \subset T$	Time indices such that startup by device j in startup state f in time interval t implies online in some interval in T_{jft}^{sus}
$T_w^{\mathrm{su,max}} \subset T$	Time indices incident to maximum startups constraint w

Table 5 continued

Symbol	Description
$T_w^{\text{en,max}} \subset T$	Time indices incident to maximum energy constraint w
$T_w^{\text{en,min}} \subset T$	Time indices incident to minimum energy constraint w
$W_j^{\text{en,max}} \subset W$	Multi-interval maximum energy constraints for device j
$W_j^{\text{en,min}} \subset W$	Multi-interval minimum energy constraints for device j
$W_j^{\text{su,max}} \subset W$	Multi-interval maximum startups constraints for device j

Table 6: Special set elements

Symbol	Description
$i_j \in I$	Connection bus of non-branch device $j \in J \setminus J^{\text{br}}$
$i_j^{\text{fr}} \in I$	From bus of branch $j \in J^{\text{br}}$.
$i_j^{\text{to}} \in I$	To bus of branch $j \in J^{\text{br}}$.
$j_k^{\text{out}} \in J$	The unique branch device outaged by contingency k
$t^{\text{end}} \in T$	Last time interval t.
$t^{\text{start}} \in T$	First time interval t.

Table 7: Parameters

Symbol	Description
a_t^{end}	End time of time interval t (h)
a_t^{mid}	Midpoint of time interval t (h)
a_t^{start}	Start time of time interval t (h)
$a_w^{\text{en,max,start}}$	Start time of multi-interval maximum energy constraint w (h)
$a_w^{\text{en,max,end}}$	End time of multi-interval maximum energy constraint w (h)
$a_w^{\text{en,min,start}}$	Start time of multi-interval minimum energy constraint w (h)
$a_w^{\text{en,min,end}}$	End time of multi-interval minimum energy constraint w (h)
$a_w^{\text{su,max,start}}$	Start time of multi-interval maximum startups constraint w (h)
$a_w^{\text{su,max,end}}$	End time of multi-interval maximum startups constraint w (h)

Table 7 continued

Symbol	Description
b_j^{ch}	Charging susceptance of AC branch j (pu)
b_j^{fr}	Shunt susceptance to ground of AC branch j at from bus (pu)
b_j^{sh}	Susceptance of one step of shunt j (pu)
b_j^{sr}	Series susceptance of AC branch j (pu)
b_j^{to}	Shunt susceptance to ground of AC branch j at to bus (pu)
c^{e}	Penalty coefficient on energy excess or shortfall in dispatchable device constraints on total energy over multiple time intervals ($/pu-h)
c_{jtm}^{en}	Energy marginal cost or value of offer or bid block m of producing or consuming device j in interval t ($/pu-h)
c^{p}	Real power bus imbalance marginal cost ($/pu-h)
c^{q}	Reactive power bus imbalance marginal cost ($/pu-h)
c_{jt}^{rgd}	Marginal cost of regulation down provided by device j in interval t ($/pu-h)
c_{jt}^{rgu}	Marginal cost of regulation up provided by device j in interval t ($/pu-h)
c_{jt}^{scr}	Marginal cost of synchronized reserve provided by device j in interval t ($/pu-h)
c_{jt}^{nsc}	Marginal cost of non-synchronized reserve provided by device j ($/pu-h) in interval t
$c_{jt}^{\mathrm{rru,off}}$	Marginal cost of ramping reserve up provided by device j when offline in interval t ($/pu-h)
$c_{jt}^{\mathrm{rru,on}}$	Marginal cost of ramping reserve up provided by device j when online in interval t ($/pu-h)
$c_{jt}^{\mathrm{rrd,off}}$	Marginal cost of ramping reserve down provided by device j when offline in interval t ($/pu-h)
$c_{jt}^{\mathrm{rrd,on}}$	Marginal cost of ramping reserve down provided by device j when online in interval t ($/pu-h)
c_{jt}^{qru}	Marginal cost of reactive power up reserve provided by device j in interval t ($/pu-h)
c_{jt}^{qrd}	Marginal cost of reactive power down reserve provided by device j in interval t ($/pu-h)
c_n^{rgu}	Marginal cost of shortfall of regulation up in zone n ($/pu-h)
c_n^{rgd}	Marginal cost of shortfall of regulation down in zone n ($/pu-h)
c_n^{scr}	Marginal cost of shortfall of synchronized reserve in zone n ($/pu-h)
c_n^{nsc}	Marginal cost of shortfall of non-synchronized reserve in zone n ($/pu-h)

Table 7 continued

Symbol	Description
c_n^{rru}	Marginal cost of shortfall of ramping reserve up in zone n (\$/pu-h)
c_n^{rrd}	Marginal cost of shortfall of ramping reserve down in zone n (\$/pu-h)
c_n^{qru}	Marginal cost of shortfall of reactive power reserve up in zone n (\$/pu-h)
c_n^{qrd}	Marginal cost of shortfall of reactive power reserve down in zone n (\$/pu-h)
c^{s}	AC branch overload marginal cost (\$/pu-h)
c_j^{on}	Fixed cost of online status of device j (\$/h)
c_j^{su}	Startup cost of device j (\$)
c_{jf}^{sus}	Downtime-dependent startup state cost of device j in startup state f (\$)
c_j^{sd}	Shutdown cost of device j (\$)
d_t	Duration of interval t (h)
$d_{jf}^{\text{dn,max}}$	Maximum prior downtime of device j if starting up in startup state f (h)
$d_j^{\text{dn,min}}$	Minimum downtime of device j (h)
$d_j^{\text{dn,0}}$	Prior downtime of device j at the start of the model horizon (h)
d^{unit}	Minimum time unit - all time durations in the model are integer multiples of this value (h)
$d_j^{\text{up,min}}$	Minimum uptime of device j (h)
$d_j^{\text{up,0}}$	Prior uptime of device j at the start of the model horizon (h)
$e_w^{\max}$	Maximum energy for multi-interval maximum energy constraint w (pu-h)
$e_w^{\min}$	Minimum energy for multi-interval minimum energy constraint w (pu-h)
g_j^{fr}	Shunt conductance to ground of AC branch j at from bus (pu)
g_j^{sr}	Series conductance of AC branch j (pu)
g_j^{sh}	Conductance of one step of shunt j (pu)
g_j^{to}	Shunt conductance to ground of AC branch j at to bus (pu)
$p_j^{\text{dc,max}}$	Maximum real power flow of DC line j (pu)
$p_{jtm}^{\max}$	Maximum real power of offer or bid block m of producing or consuming device j in interval t (pu)
$p_{jt}^{\max}$	Maximum real power of producing or consuming device j when online in interval t (pu)
$p_{jt}^{\min}$	Minimum real power of producing or consuming device j when online in interval t (pu)

Table 7 continued

Symbol	Description
p_j^{rd}	Maximum ramp down rate of producing or consuming device j when online (pu/h)
$p_j^{\text{rd,sd}}$	Maximum ramp down rate of producing or consuming device j when shutting down (pu/h)
p_j^{ru}	Maximum ramp up rate of producing or consuming device j when online and not starting up (pu/h)
$p_j^{\text{ru,su}}$	Maximum ramp up rate of producing or consuming device j when starting up (pu/h)
$p_{jtt'}^{\text{supc}}$	Startup power of producing or consuming device j in interval t if starting up in interval $t' > t$ in its startup power curve (pu)
$p_{jtt'}^{\text{sdpc}}$	Shutdown power of producing or consuming device j in interval t if shutting down in interval $t' \leq t$ in its shutdown power curve (pu)
p_j^0	Initial real power of producing or consuming device j (pu)
$p_j^{\text{rgu,max}}$	Maximum regulation up for producing or consuming device j (pu)
$p_j^{\text{rgd,max}}$	Maximum regulation down for producing or consuming device j (pu)
$p_j^{\text{scr,max}}$	Maximum synchronized reserve for producing or consuming device j (pu)
$p_j^{\text{nsc,max}}$	Maximum non-synchronized reserve for producing or consuming device j (pu)
$p_j^{\text{rru,on,max}}$	Maximum ramping reserve up for producing or consuming device j when online (pu)
$p_j^{\text{rru,off,max}}$	Maximum ramping reserve up for producing or consuming device j when offline (pu)
$p_j^{\text{rrd,on,max}}$	Maximum ramping reserve down for producing or consuming device j when online (pu)
$p_j^{\text{rrd,off,max}}$	Maximum ramping reserve down for producing or consuming device j when offline (pu)
$p_{nt}^{\text{rru,min}}$	Exogenous ramping reserve up requirement for zone n in interval t (pu)
$p_{nt}^{\text{rrd,min}}$	Exogenous ramping reserve down requirement for zone n in interval t (pu)
q_j^{q0}	Prior value of reactive power of producing or consuming device j (pu)
q_j^{p0}	Reactive power at 0 real power of producing or consuming device j if the device has reactive power dependent on real power (pu)
$q_j^{\text{max,p0}}$	Upper bound on reactive power at 0 real power of producing or consuming device j if the device has maximum reactive power dependent on real power (pu)

Table 7 continued

Symbol	Description
$q_j^{\text{min,p0}}$	Lower bound on reactive power at 0 real power of producing or consuming device j if the device has minimum reactive power dependent on real power (pu)
q_{jt}^{max}	Maximum reactive power of producing or consuming device j when online in interval t (pu)
q_{jt}^{min}	Minimum reactive power of producing or consuming device j when online in interval t (pu)
$q_j^{\text{dc,fr,max}}$	Maximum reactive power at from bus of DC line j (pu)
$q_j^{\text{dc,to,max}}$	Maximum reactive power at to bus of DC line j (pu)
$q_j^{\text{dc,fr,min}}$	Minimum reactive power at from bus of DC line j (pu)
$q_j^{\text{dc,to,min}}$	Minimum reactive power at to bus of DC line j (pu)
$q_{nt}^{\text{qru,min}}$	Exogenous reactive power reserve up requirement for zone n in interval t (pu)
$q_{nt}^{\text{qrd,min}}$	Exogenous reactive power reserve down requirement for zone n in interval t (pu)
r_j^{sr}	Series resistance of AC branch j (pu)
s_j^{max}	Maximum apparent power flow of branch j (pu)
$s_j^{\text{max,ctg}}$	Maximum apparent power flow of branch j in contingencies (pu)
$u_{jt}^{\text{on,max}}$	Upper bound on online status indicator for device j in interval t (binary)
$u_{jt}^{\text{on,min}}$	Lower bound on online status indicator for device j in interval t (binary)
$u_j^{\text{on,0}}$	Initial on-off status of device j (binary)
$u_w^{\text{su,max}}$	Maximum startups for multi-interval maximum startups constraint w (integer)
$u_j^{\text{sh,0}}$	Prior number of activated steps of shunt j (integer)
$u_j^{\text{sh,max}}$	Maximum number of activated steps of shunt j (integer)
$u_j^{\text{sh,min}}$	Minimum number of activated steps of shunt j (integer)
v_i^0	Prior value of voltage magnitude at bus i (pu)
v_i^{max}	Maximum voltage magnitude at bus i (pu)
v_i^{min}	Minimum voltage magnitude at bus i (pu)
x_j^{sr}	Series reactance of AC branch j (pu)

Table 7 continued

Symbol	Description
α_i	Participation factor of bus i in resolving system real power imbalance in contingencies (dimensionless)
β_j	Sensitivity of reactive power with respect to real power of producing or consuming device j with equality constraint linking real and reactive power (pu/pu)
$\beta_j^{\max}$	Sensitivity of upper bound on reactive power with respect to real power of producing or consuming device j with inequality constraint modeling upper bounds on reactive power depending on real power (pu/pu)
$\beta_j^{\min}$	Sensitivity of lower bound on reactive power with respect to real power of producing or consuming device j with inequality constraint modeling upper bounds on reactive power depending on real power (pu/pu)
ϵ^{beta}	Beta-value cutoff. Beta coefficients less than this value are prohibited. (pu/pu)
ϵ^{constr}	Hard constraint feasibility tolerance. Violations of hard constraints are allowed (deemed feasible) up to this tolerance and not allowed (deemed infeasible) beyond it. (units are those of the constraint where the tolerance is applied, generally pu, rad, or dimensionless)
ϵ^{int}	Integrality tolerance. This tolerance is used to evaluate whether derived data items that need to be integers are close enough to integers. It is not used as an integrality tolerance on integer variables in the solution. Integer variables in the solution need to be written in the solution file as exact integers. (dimensionless)
ϵ^{susd}	Startup/shutdown trajectory p-value cutoff. Values less than this are considered to be ambiguous. That is, it is not clear if such a value is in the trajectory or not. Therefore, the data is constructed to avoid this. (pu)
ϵ^{time}	Tolerance for comparing time quantities. In constructing the derived data, two time durations differing by less than this value are treated as equal, in order to account for floating point arithmetic in their construction. (h)
σ_n^{rgd}	Fraction of total cleared power consumption in zone n forming regulation down requirement for zone n (dimensionless)
σ_n^{rgu}	Fraction of total cleared power consumption in zone n forming regulation up requirement for zone n (dimensionless)
σ_n^{scr}	Fraction of largest cleared power production in zone n forming synchronized reserve requirement for zone n (dimensionless)
σ_n^{nsc}	Fraction of largest cleared power production in zone n forming non-synchronized reserve requirement for zone n (dimensionless)
τ_j^0	Prior value of winding ratio of AC branch j (dimensionless)

Table 7 continued

Symbol	Description
$\tau_j^{\max}$	Maximum winding ratio of AC branch j (dimensionless)
$\tau_j^{\min}$	Minimum winding ratio of AC branch j (dimensionless)
θ_i^0	Prior value of voltage angle at bus i. (rad)
ϕ_j^0	Prior value of phase difference of AC branch j (rad)
$\phi_j^{\max}$	Maximum phase difference of AC branch j (rad)
$\phi_j^{\min}$	Minimum phase difference of AC branch j (rad)

Table 8: Variables

Symbol	Description
b_{jt}^{sh}	Susceptance of shunt j in interval t. (pu)
e_w^+	Excess or shortfall of energy in maximum or minimum energy constraint w covering total energy of a single dispatchable device over multiple time intervals. (pu-h)
g_{jt}^{sh}	Conductance of shunt j in interval t. (pu)
p_{it}	Real power signed mismatch, i.e. load and other real power consumption and absorption by branches and shunts, minus generation and other production, at bus i in interval t. (pu)
p_{it}^+	Real power penalized mismatch, i.e. absolute value of signed mismatch, at bus i in interval t. (pu)
p_{jt}	Real power of device j in interval t. Oriented from the device into the connection bus for producing devices and from the connection bus into the device for consuming devices and shunts. (pu)
p_{jtk}	Real power of device j in interval t contingency k. Oriented from the device into the connection bus for dispatchable devices, from the device into the connection bus for non-dispatchable devices, and from the from bus to the to bus for branches. (pu)
p_{jt}^{on}	Dispatchable real power of producing or consuming device j in interval t. (pu)
p_{jt}^{su}	Startup real power of producing or consuming device j in interval t. (pu)
p_{jt}^{sd}	Shutdown real power of producing or consuming device j in interval t. (pu)
p_{jtm}	Real power of producing or consuming device j in interval t in energy cost or value block m. (pu)

Table 8 continued

Symbol	Description
p_{jt}^{fr}	Real power at from bus of branch j in interval t. Oriented from the bus into the device. (pu)
p_{jt}^{rgu}	Regulation up provided by device j in interval t
p_{jt}^{rgd}	Regulation down provided by device j in interval t
p_{jt}^{scr}	Synchronized reserve provided by device j in interval t
p_{jt}^{nsc}	Non-synchronized reserve provided by device j in interval t
$p_{jt}^{\text{rru,on}}$	Ramping reserve up provided by device j in interval t when online
$p_{jt}^{\text{rru,off}}$	Ramping reserve up provided by device j in interval t when offline
$p_{jt}^{\text{rrd,on}}$	Ramping reserve down provided by device j in interval t when online
$p_{jt}^{\text{rrd,off}}$	Ramping reserve down provided by device j in interval t when offline
$p_{nt}^{\text{rgu,req}}$	Regulation up requirement for reserve zone n in interval t
$p_{nt}^{\text{rgd,req}}$	Regulation down requirement for reserve zone n in interval t
$p_{nt}^{\text{scr,req}}$	Synchronized reserve requirement for reserve zone n in interval t
$p_{nt}^{\text{nsc,req}}$	Non-synchronized reserve requirement for reserve zone n in interval t
$p_{nt}^{\text{rgu},+}$	Regulation up shortfall for reserve zone n in interval t
$p_{nt}^{\text{rgd},+}$	Regulation down shortfall for reserve zone n in interval t
$p_{nt}^{\text{scr},+}$	Synchronized reserve shortfall for reserve zone n in interval t
$p_{nt}^{\text{nsc},+}$	Non-synchronized reserve shortfall for reserve zone n in interval t
$p_{nt}^{\text{rru},+}$	Ramping reserve up shortfall for reserve zone n in interval t
$p_{nt}^{\text{rrd},+}$	Ramping reserve down shortfall for reserve zone n in interval t
p_{jt}^{to}	Real power at to bus of branch j in interval t. Oriented from the bus into the device. (pu)
p_{t}^{sl}	Real power system slack in interval t, positive corresponds to net positive losses in the branches (pu)
q_{it}	Reactive power signed mismatch, i.e. load and other reactive power consumption and absorption by branches and shunts, minus generation and other production, at bus i in interval t. (pu)
q_{it}^{+}	Reactive power penalized mismatch, i.e. absolute value of signed mismatch, at bus i in interval t. (pu)
q_{jt}	Reactive power of non-branch device j in interval t. Oriented from the device into the connection bus for producing devices and from the connection bus into the device for consuming devices and shunts. (pu)

Table 8 continued

Symbol	Description
q^{fr}_{jt}	Reactive power at from bus of branch j in interval t. Oriented from the bus into the device. (pu)
q^{to}_{jt}	Reactive power at to bus of branch j in interval t. Oriented from the bus into the device. (pu)
q^{qru}_{jt}	Reactive power up reserve provided by device j in interval t
q^{qrd}_{jt}	Reactive power down reserve provided by device j in interval t
$q^{\text{qru},+}_{nt}$	Reactive power up reserve shortfall for reserve zone n in interval t
$q^{\text{qrd},+}_{nt}$	Reactive power down reserve shortfall for reserve zone n in interval t
s^{+}_{jt}	AC branch j apparent power overload in interval t. (pu)
s^{+}_{jtk}	AC branch j (approximate) apparent overload in interval t contingency k. (pu)
u^{on}_{jt}	On indicator of device j in interval t. The value 1 indicates the device is online or closed, 0 else. (binary)
u^{sd}_{jt}	Shutdown indicator of device j in interval t. The value 1 indicates the device is shutting down, transitioning from online to offline or from closed to open, 0 else. (binary)
u^{sh}_{jt}	Number of steps activated for shunt j in interval t. (integer)
u^{su}_{jt}	Startup indicator of device j in interval t. The value 1 indicates the device is starting up, transitioning from offline to online or from open to closed, 0 else. (binary)
u^{sus}_{jft}	Downtime-dependent startup state indicator of device j in interval t state f. (binary)
v_{it}	Voltage magnitude at bus i in interval t. (pu)
z^{ms}	Total maximization objective, i.e. base case objective, plus worst case post-contingency objective, plus average case post-contingency objective. Net market surplus, including values minus costs and penalties. ($)
z^{base}	Total base case objective. ($)
$z^{\text{ctg,avg}}$	Average case post-contingency objective. ($)
$z^{\text{ctg,min}}$	Worst case post-contingency objective. ($)
$z^{\text{ctg,avg}}_{t}$	Average case post-contingency objective in interval t. ($)
$z^{\text{ctg,min}}_{t}$	Worst case post-contingency objective in interval t. ($)
z^{ctg}_{tk}	Post-contingency objective in interval t contingency k. ($)
z^{t}_{t}	Base case time-indexed objective in interval t. ($)
$z^{\text{en,max}}_{w}$	Base case constraint-indexed objective for maximum energy constraint w. ($)

Table 8 continued

Symbol	Description
$z_w^{\mathrm{en,min}}$	Base case constraint-indexed objective for minimum energy constraint w. (\$)
z_{jt}^{en}	Energy cost or value of producing or consuming device j in interval t. (\$)
$z_w^{\mathrm{en,max}}$	Penalty on violation of maximum energy constraint w covering a single dispatchable device over multiple time intervals. (\$)
$z_w^{\mathrm{en,min}}$	Penalty on violation of minimum energy constraint w covering a single dispatchable device over multiple time intervals. (\$)
z_{jt}^{on}	Online status cost of device j in interval t. (\$)
z_{it}^{p}	Real power mismatch cost of bus i in interval t. (\$)
z_{it}^{q}	Reactive power mismatch cost of bus i in interval t. (\$)
z_{jt}^{sd}	Shutdown cost of device j in interval t. (\$)
z_{jt}^{su}	Startup cost of device j in interval t. (\$)
z_{jt}^{sus}	Downtime-dependent startup cost of device j in interval t. (\$)
z_{jt}^{s}	AC branch overload cost of AC branch j in interval t. (\$)
z_{jtk}^{s}	AC branch overload cost of AC branch j in interval t contingency k. (\$)
z_{jt}^{rgu}	Cost of Regulation up provided by device j in interval t (\$)
z_{jt}^{rgd}	Cost of Regulation down provided by device j in interval t (\$)
z_{jt}^{scr}	Cost of Synchronized reserve provided by device j in interval t and not counting as regulation up (\$)
z_{jt}^{nsc}	Cost of Non-synchronized reserve provided by device j in interval t (\$)
z_{jt}^{rru}	Cost of ramping reserve up provided by device j in interval t (\$)
z_{jt}^{rrd}	Cost of ramping reserve down provided by device j in interval t (\$)
z_{jt}^{qru}	Cost of Reactive power up reserve provided by device j in interval t (\$)
z_{jt}^{qrd}	Cost of Reactive power down reserve provided by device j in interval t (\$)
z_{nt}^{rgu}	Regulation up shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{rgd}	Regulation down shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{scr}	Synchronized reserve shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{nsc}	Non-synchronized reserve shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{rru}	Ramping reserve up shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{rrd}	Ramping reserve down shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{qru}	Reactive power up reserve shortfall penalty for reserve zone n in interval t (\$)
z_{nt}^{qrd}	Reactive power down reserve shortfall penalty for reserve zone n in interval t (\$)

Table 8 continued

Symbol	Description
θ_{it}	Voltage angle at bus i in interval t. (rad)
θ_{itk}	Approximate post-contingency voltage angle at bus i in interval t in contingency k. (rad)
τ_{jt}	Winding ratio of AC branch j in interval t. (dimensionless)
ϕ_{jt}	Phase difference of AC branch j in interval t. (rad)

4 Optimization model formulation

This section formulates the optimization model. The model formulation consists of a set of constraints involving variables and parameters, structured by index sets, subsets, and special set elements, together with an objective specified as one of the variables, and a specification of a direction of optimization, specifically that the objective should be maximimized. This section formulates these constraints as algebraic equations and introduces the variables. The variables and constraints are presented in an ordered and hierarchical structure with the goals of comprehensibility, completeness, accuracy, and precision. This model formulation is called the reference formulation.

4.1 Market surplus objective

The objective z^{ms} for maximization is the total market surplus, consisting of the base case market surplus z^{base} plus terms $z^{\text{ctg,min}}$ and $z^{\text{ctg,avg}}$ covering the worst case and average case of post-contingency outcomes:

$$z^{\text{ms}} = z^{\text{base}} + z^{\text{ctg,min}} + z^{\text{ctg,avg}} \tag{1}$$

The worst case and average case post-contingency objective terms are each the sum over time intervals of corresponding post-contingency objectives $z_t^{\text{ctg,min}}$ and $z_t^{\text{ctg,avg}}$ for each interval:

$$z^{\text{ctg,min}} = \sum_{t \in T} z_t^{\text{ctg,min}} \tag{2}$$

$$z^{\text{ctg,avg}} = \sum_{t \in T} z_t^{\text{ctg,avg}} \tag{3}$$

For each interval, the worst case and average case post-contingency objective terms are defined as the minimum and the average over contingencies of post-contingency z_{tk}^{ctg} for each contingency, and if $|K|= 0$, then these terms are interpreted as 0:

$$z_t^{\text{ctg,min}} = \min_{k \in K} z_{tk}^{\text{ctg}} \text{ if } |K|> 0 \text{ else } 0 \ \forall t \in T \tag{4}$$

$$z_t^{\text{ctg,avg}} = 1/|K| \sum_{k \in K} z_{tk}^{\text{ctg}} \text{ if } |K|> 0 \text{ else } 0 \ \forall t \in T \tag{5}$$

The base case objective term is the sum over intervals of the base case objective z^{t}_{t} for each interval plus the sum over producing and consuming devices of objective terms $z^{\mathrm{en,max}}_{j}$ and $z^{\mathrm{en,min}}_{j}$ for maximum and minimum energy constraint violations that cannot be indexed to time intervals:

$$z^{\mathrm{base}} = \sum_{t \in T} z^{\mathrm{t}}_{t} + \sum_{j \in J^{\mathrm{pr,cs}}} z^{\mathrm{en,max}}_{j} + \sum_{j \in J^{\mathrm{pr,cs}}} z^{\mathrm{en,min}}_{j} \tag{6}$$

The maximum and minimum energy constraint violation terms are the sums over individual constraints of violation penalties $z^{\mathrm{en,max}}_{w}$ and $z^{\mathrm{en,min}}_{w}$, with negative signs:

$$z^{\mathrm{en,max}}_{j} = - \sum_{w \in W^{\mathrm{en,max}}_{j}} z^{\mathrm{en,max}}_{w} \; \forall j \in J^{\mathrm{pr,cs}} \tag{7}$$

$$z^{\mathrm{en,min}}_{j} = - \sum_{w \in W^{\mathrm{en,min}}_{j}} z^{\mathrm{en,min}}_{w} \; \forall j \in J^{\mathrm{pr,cs}} \tag{8}$$

The time-indexed base case market surplus z^{t}_{t} is the sum of appropriately signed terms representing values accrued and costs incurred by consumers, producers, device startup, shutdown, and fixed operating cost, branch limit overload penalties, device reserve procurement, bus real and reactive power mismatch penalties, and zonal reserve shortfall penalties:

$$\begin{aligned} z^{\mathrm{t}}_{t} = & \sum_{j \in J^{\mathrm{cs}}} z^{\mathrm{en}}_{jt} - \sum_{j \in J^{\mathrm{pr}}} z^{\mathrm{en}}_{jt} - \sum_{j \in J^{\mathrm{pr,cs,ac}}} \left(z^{\mathrm{su}}_{jt} + z^{\mathrm{sd}}_{jt} \right) - \sum_{j \in J^{\mathrm{pr,cs}}} \left(z^{\mathrm{on}}_{jt} + z^{\mathrm{sus}}_{jt} \right) - \sum_{j \in J^{\mathrm{ac}}} z^{\mathrm{s}}_{jt} \\ & - \sum_{j \in J^{\mathrm{pr,cs}}} \left(z^{\mathrm{rgu}}_{jt} + z^{\mathrm{rgd}}_{jt} + z^{\mathrm{scr}}_{jt} + z^{\mathrm{nsc}}_{jt} + z^{\mathrm{rru}}_{jt} + z^{\mathrm{rrd}}_{jt} + z^{\mathrm{qru}}_{jt} + z^{\mathrm{qrd}}_{jt} \right) - \sum_{i \in I} \left(z^{\mathrm{p}}_{it} + z^{\mathrm{q}}_{it} \right) \\ & - \sum_{n \in N^{\mathrm{p}}} \left(z^{\mathrm{rgu}}_{nt} + z^{\mathrm{rgd}}_{nt} + z^{\mathrm{scr}}_{nt} + z^{\mathrm{nsc}}_{nt} + z^{\mathrm{rru}}_{nt} + z^{\mathrm{rrd}}_{nt} \right) - \sum_{n \in N^{\mathrm{q}}} \left(z^{\mathrm{qru}}_{nt} + z^{\mathrm{qrd}}_{nt} \right) \; \forall t \in T \end{aligned} \tag{9}$$

The post-contingency objective is the negative of the sum of branch limit overload penalties z^{s}_{jtk} on AC branches in service in each contingency:

$$z^{\mathrm{ctg}}_{tk} = - \sum_{j \in J^{\mathrm{ac}}_{k}} z^{\mathrm{s}}_{jtk} \; \forall t \in T, k \in K \tag{10}$$

4.2 Bus real and reactive power balance and voltage

4.2.1 Bus power mismatch and penalized mismatch definitions

For each bus i and each interval t, the penalized real and reactive power mismatches p^{+}_{it} and q^{+}_{it} are the absolute values of the signed mismatches p_{it} and q_{it}:

$$p^{+}_{it} \geq p_{it} \; \forall t \in T, i \in I \tag{11}$$

$$p^{+}_{it} \geq -p_{it} \; \forall t \in T, i \in I \tag{12}$$

$$q^{+}_{it} \geq q_{it} \; \forall t \in T, i \in I \tag{13}$$

$$q^{+}_{it} \geq -q_{it} \; \forall t \in T, i \in I \tag{14}$$

4.2.2 Bus power mismatch penalty

For each bus i and interval t the bus real and reactive power mismatch penalties z^{p}_{it} and z^{q}_{it} are defined by linear penalty functions applied to the penalized mismatches:

$$z^{\text{p}}_{it} = d_t c^{\text{p}} p^{+}_{it} \ \forall t \in T, i \in I \tag{15}$$

$$z^{\text{q}}_{it} = d_t c^{\text{q}} q^{+}_{it} \ \forall t \in T, i \in I \tag{16}$$

4.2.3 Bus real and reactive power balance

For each bus i and each interval t, real and reactive power balance are enforced by:

$$\sum_{j \in J^{\text{cs}}_i} p_{jt} + \sum_{j \in J^{\text{sh}}_i} p_{jt} + \sum_{j \in J^{\text{fr}}_i} p^{\text{fr}}_{jt} + \sum_{j \in J^{\text{to}}_i} p^{\text{to}}_{jt} = \sum_{j \in J^{\text{pr}}_i} p_{jt} + p_{it} \ \forall t \in T, i \in I \tag{17}$$

$$\sum_{j \in J^{\text{cs}}_i} q_{jt} + \sum_{j \in J^{\text{sh}}_i} q_{jt} + \sum_{j \in J^{\text{fr}}_i} q^{\text{fr}}_{jt} + \sum_{j \in J^{\text{to}}_i} q^{\text{to}}_{jt} = \sum_{j \in J^{\text{pr}}_i} q_{jt} + q_{it} \ \forall t \in T, i \in I \tag{18}$$

In Eqs. (17) and (18) power withdrawals at bus i in interval t appear on the left, and injections appear on the right. Withdrawals include consumption by consuming devices, signed absorption by shunts, signed flow directed into branches at the from and to buses. Injections include production by producing devices and signed power mismatch.

4.2.4 Bus voltage

For each bus i and each interval t, the voltage angle and magnitude are represented by variables θ_{it} and v_{it}. Limits on voltage magnitude are modeled by hard constraints:

$$v^{\min}_i \le v_{it} \le v^{\max}_i \ \forall t \in T, i \in I \tag{19}$$

4.3 Zonal reserve requirements

4.3.1 Reserve shortfall domains

For each reserve product, in each reserve zone n and time interval t, a nonnegative variable represents the shortfall of procured reserve relative to the reserve requirement:

$$p^{\text{rgu},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{p}} \tag{20}$$

$$p^{\text{rgd},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{p}} \tag{21}$$

$$p^{\text{scr},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{p}} \tag{22}$$

$$p^{\text{nsc},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{p}} \tag{23}$$

$$p^{\text{rru},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{p}} \tag{24}$$

$$p^{\text{rrd},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{p}} \tag{25}$$

$$q^{\text{qru},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{q}} \tag{26}$$

$$q^{\text{qrd},+}_{nt} \ge 0 \ \forall t \in T, n \in N^{\text{q}} \tag{27}$$

4.3.2 Reserve shortfall penalties

Reserve shortfall penalties are defined by a linear penalty function applied the reserve shortfall, expressed as the interval duration times the penalty coefficient times the shortfall:

$$z_{nt}^{\text{rgu}} = d_t c_n^{\text{rgu}} p_{nt}^{\text{rgu},+} \ \forall t \in T, n \in N^{\text{p}} \tag{28}$$

$$z_{nt}^{\text{rgd}} = d_t c_n^{\text{rgd}} p_{nt}^{\text{rgd},+} \ \forall t \in T, n \in N^{\text{p}} \tag{29}$$

$$z_{nt}^{\text{scr}} = d_t c_n^{\text{scr}} p_{nt}^{\text{scr},+} \ \forall t \in T, n \in N^{\text{p}} \tag{30}$$

$$z_{nt}^{\text{nsc}} = d_t c_n^{\text{nsc}} p_{nt}^{\text{nsc},+} \ \forall t \in T, n \in N^{\text{p}} \tag{31}$$

$$z_{nt}^{\text{rru}} = d_t c_n^{\text{rru}} p_{nt}^{\text{rru},+} \ \forall t \in T, n \in N^{\text{p}} \tag{32}$$

$$z_{nt}^{\text{rrd}} = d_t c_n^{\text{rrd}} p_{nt}^{\text{rrd},+} \ \forall t \in T, n \in N^{\text{p}} \tag{33}$$

$$z_{nt}^{\text{qru}} = d_t c_n^{\text{qru}} q_{nt}^{\text{qru},+} \ \forall t \in T, n \in N^{\text{q}} \tag{34}$$

$$z_{nt}^{\text{qrd}} = d_t c_n^{\text{qrd}} q_{nt}^{\text{qrd},+} \ \forall t \in T, n \in N^{\text{q}} \tag{35}$$

4.3.3 Reserve requirements

For certain products, specifically ramping reserve up and down and reactive power reserve up and down, the reserve requirement is an exogenous value given by input data, and this value is used directly in the reserve balance constraints. For the other products, the requirement is an endogenous value formed in a prescribed fashion from the power dispatch. The endogenous reserve requirements are given by:

$$p_{nt}^{\text{rgu,req}} = \sigma_n^{\text{rgu}} \sum_{j \in J_n^{\text{cs}}} p_{jt} \ \forall t \in T, n \in N^{\text{p}} \tag{36}$$

$$p_{nt}^{\text{rgd,req}} = \sigma_n^{\text{rgd}} \sum_{j \in J_n^{\text{cs}}} p_{jt} \ \forall t \in T, n \in N^{\text{p}} \tag{37}$$

$$p_{nt}^{\text{scr,req}} = \sigma_n^{\text{scr}} \max_{j \in J_n^{\text{pr}}} p_{jt} \ \forall t \in T, n \in N^{\text{p}} \tag{38}$$

$$p_{nt}^{\text{nsc,req}} = \sigma_n^{\text{nsc}} \max_{j \in J_n^{\text{pr}}} p_{jt} \ \forall t \in T, n \in N^{\text{p}} \tag{39}$$

The requirements for regulation up and down are modeled endogenously in Eqs. (36) and (37) as prescribed factors of the total real power over all consuming devices. The requirements for synchronized and non-synchronized reserves are modeled endogenously in Eqs. (38) and (39) as a prescribed factors of the largest real power over all producing devices. In the event that $J_n^{\text{pr}} = \{\}$ for some $n \in N^{\text{p}}$, the max operator in Eqs. (38) and (39) is interpreted as taking the value 0.

4.3.4 Reserve balance

For each reserve product, in each zone n and each interval t, the total reserve procured from producing and consuming devices plus any shortfall must be greater than or equal to the requirement. Excess of higher quality reserve products can substitute for lower quality products, as modeled in Eqs. (42) and (43). Some products can be provided by online devices and by offline devices and are tracked by separate variables, as in Eqs. (44) and (45).

$$\sum_{j \in J_n^{\text{pr,cs}}} p_{jt}^{\text{rgu}} + p_{nt}^{\text{rgu,+}} \geq p_{nt}^{\text{rgu,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{40}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} p_{jt}^{\text{rgd}} + p_{nt}^{\text{rgd,+}} \geq p_{nt}^{\text{rgd,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{41}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} \left(p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}}\right) + p_{nt}^{\text{scr,+}} \geq p_{nt}^{\text{rgu,req}} + p_{nt}^{\text{scr,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{42}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} \left(p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} + p_{jt}^{\text{nsc}}\right) + p_{nt}^{\text{nsc,+}} \geq p_{nt}^{\text{rgu,req}} + p_{nt}^{\text{scr,req}} + p_{nt}^{\text{nsc,req}} \ \forall t \in T, n \in N^{\text{p}} \tag{43}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} \left(p_{jt}^{\text{rru,on}} + p_{jt}^{\text{rru,off}}\right) + p_{nt}^{\text{rru,+}} \geq p_{nt}^{\text{rru,min}} \ \forall t \in T, n \in N^{\text{p}} \tag{44}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} \left(p_{jt}^{\text{rrd,on}} + p_{jt}^{\text{rrd,off}}\right) + p_{nt}^{\text{rrd,+}} \geq p_{nt}^{\text{rrd,min}} \ \forall t \in T, n \in N^{\text{p}} \tag{45}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} q_{jt}^{\text{qru}} + q_{nt}^{\text{qru,+}} \geq q_{nt}^{\text{qru,min}} \ \forall t \in T, n \in N^{\text{q}} \tag{46}$$

$$\sum_{j \in J_n^{\text{pr,cs}}} q_{jt}^{\text{qrd}} + q_{nt}^{\text{qrd,+}} \geq q_{nt}^{\text{qrd,min}} \ \forall t \in T, n \in N^{\text{q}} \tag{47}$$

4.4 Device on-off status and related constraints

4.4.1 Device on-off status

For each device j and each interval t, a binary variable u_{jt}^{on} represents the on-off status, with $u_{jt}^{\text{on}} = 1$ if device j is online in interval t and 0 otherwise. Binary variables u_{jt}^{su} and u_{jt}^{sd} are also used to indicate startup and shutdown.

$$u_{jt}^{\text{on}} \in \{0, 1\} \ \forall t \in T, j \in J^{\text{pr,cs,ac}} \tag{48}$$

$$u_{jt}^{\text{su}} \in \{0, 1\} \ \forall t \in T, j \in J^{\text{pr,cs,ac}} \tag{49}$$

$$u_{jt}^{\text{sd}} \in \{0, 1\} \ \forall t \in T, j \in J^{\text{pr,cs,ac}} \tag{50}$$

Device on-off statuses are subject to planned outage and must-run conditions:

$$u_{jt}^{\text{on}} = 1 \ \forall j \in J^{\text{pr,cs}}, t \in T_j^{\text{mr}} \tag{51}$$

$$u_{jt}^{\text{on}} = 0 \ \forall j \in J^{\text{pr,cs}}, t \in T_j^{\text{out}} \tag{52}$$

On-off status, startup, and shutdown variables are related by evolution equations:

$$u_{jt}^{\text{on}} - u_j^{\text{on,0}} = u_{jt}^{\text{su}} - u_{jt}^{\text{sd}} \ \forall t = t^{\text{start}}, j \in J^{\text{pr,cs,ac}} \tag{53}$$

$$u_{jt}^{\text{on}} - u_{j,t-1}^{\text{on}} = u_{jt}^{\text{su}} - u_{jt}^{\text{sd}} \ \forall t > t^{\text{start}}, j \in J^{\text{pr,cs,ac}} \tag{54}$$

In Eq. (53) for the first time interval, the prior on-off status $u_j^{\text{on},0}$ is a parameter from the input data. In Eq. (54) for intervals after the first one, the prior on-off status $u_{j,t-1}^{\text{on}}$ is the on-off status variable from the prior time interval. Simultaneous startup and shutdown is prohibited:

$$u_{jt}^{\text{su}} + u_{jt}^{\text{sd}} \leq 1 \ \forall t \in T, j \in J^{\text{pr,cs,ac}} \tag{55}$$

4.4.2 Minimum downtime

After a device shuts down it must remain offline for a prescribed minimum downtime before starting up. Logical constraints enforce this algebraically by requiring that a device cannot start up if it has shut down in a prescribed set of prior intervals:

$$u_{jt}^{\text{su}} \leq 1 - \sum_{t' \in T_{jt}^{\text{dn,min}}} u_{jt'}^{\text{sd}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{56}$$

4.4.3 Minimum uptime

Similar to minimum downtime, minimum uptime is enforced by a logical constraint requiring no recent startup if shutting down:

$$u_{jt}^{\text{sd}} \leq 1 - \sum_{t' \in T_{jt}^{\text{up,min}}} u_{jt'}^{\text{su}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{57}$$

4.4.4 Maximum starts over multiple intervals

Some devices have limits on the number of startups in a prescribed set of time intervals:

$$\sum_{t \in T_w^{\text{su,max}}} u_{jt}^{\text{su}} \leq u_w^{\text{su,max}} \ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{su,max}} \tag{58}$$

4.4.5 Synchronous network connectivity constraints on branch device on-off status

In each time interval, in the base case and in each contingency, the graph consisting of all buses and online in service AC branches must be connected:

$$\text{The bus-branch graph on } (I, \{j \in J^{\text{ac}} : u_{jt}^{\text{on}} = 1\}) \text{ is connected } \forall t \in T \tag{59}$$

$$\text{The bus-branch graph on } (I, \{j \in J_k^{\text{ac}} : u_{jt}^{\text{on}} = 1\}) \text{ is connected } \forall t \in T, k \in K \tag{60}$$

4.4.6 On-off status and transition costs

Online cost, startup cost, and shutdown cost are driven by the on-off status, startup, and shutdown variables:

$$z_{jt}^{\text{on}} = d_t c_j^{\text{on}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{61}$$

$$z_{jt}^{\text{su}} = c_j^{\text{su}} u_{jt}^{\text{su}} \ \forall t \in T, j \in J^{\text{pr,cs,ac}} \tag{62}$$

$$z_{jt}^{\text{sd}} = c_j^{\text{sd}} u_{jt}^{\text{sd}} \ \forall t \in T, j \in J^{\text{pr,cs,ac}} \tag{63}$$

4.4.7 Downtime-dependent startup costs

For some devices the startup cost depends on the prior downtime at the time of startup. To model this, we introduce a set of startup states F_j for device j, each characterized by a maximum prior downtime and a cost. We introduce a variable u^{sus}_{jft} indicating that device j is starting up in interval t in startup state f. The startup state indicator variables are binary:

$$u^{\text{sus}}_{jft} \in \{0, 1\} \ \forall t \in T, j \in J^{\text{pr,cs}}, f \in F_j \tag{64}$$

The downtime-dependent startup cost is

$$z^{\text{sus}}_{jt} = \sum_{f \in F_j} c^{\text{sus}}_{jf} u^{\text{sus}}_{jft} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{65}$$

If a device is in any of its startup states, then it must be starting up:

$$\sum_{f \in F_j} u^{\text{sus}}_{jft} \leq u^{\text{su}}_{jt} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{66}$$

If a device is in a startup state, then it must have been on in a recent prior interval, depending on the maximum downtime and prior downtime at the start of the model horizon:

$$u^{\text{sus}}_{jft} \leq \sum_{t' \in T^{\text{sus}}_{jft}} u^{\text{on}}_{jt'} \ \forall t \in T^{\text{sus}}_{jf}, j \in J^{\text{pr,cs}}, f \in F_j \tag{67}$$

Note, if a device is starting up with prior downtime exceeding the maximum for all of its startup states, then it is not in any startup state. The startup state cost coefficients c^{sus}_{jf} are generally negative and can be viewed as a discount on the regular startup cost that is earned by starting up after only a short prior downtime.

4.5 Device real and reactive power flow

Some devices, specifically shunts, producing devices, consuming devices, and branches, have real and reactive power flows. Shunts, producing devices, and consuming devices are each connected to a single bus, and for each such device j and each interval t, the real and reactive power flows are represented by variables p_{jt} and q_{jt}. For shunts and consuming devices, these flows are oriented so that a positive value of the flow variable represents flow directed from the bus into the device, while for producing devices, the flow is directed from the device into the bus. Each branch device j is connected to two buses, a from bus and a to bus, and the real and reactive power flows directed from the bus into the branch at the from and to buses in interval t are represented by variables p^{fr}_{jt}, q^{fr}_{jt}, p^{to}_{jt}, and q^{to}_{jt}.

4.6 Producing and consuming devices

4.6.1 Producing and consuming device startup, shutdown, and dispatchable power

Total power p_{jt} is equal to dispatchable power p^{on}_{jt}, plus startup power p^{su}_{jt}, plus shutdown power p^{sd}_{jt}:

$$p_{jt} = p^{\text{on}}_{jt} + p^{\text{su}}_{jt} + p^{\text{sd}}_{jt} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{68}$$

Startup and shutdown power are determined by linear coefficients applied to future startup and past shutdown if startup and shutdown power curves exist.

$$p^{\text{su}}_{jt} = \sum_{t' \in T^{\text{supc}}_{jt}} p^{\text{supc}}_{jtt'} u^{\text{su}}_{jt'} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{69}$$

$$p^{\text{sd}}_{jt} = \sum_{t' \in T^{\text{sdpc}}_{jt}} p^{\text{sdpc}}_{jtt'} u^{\text{sd}}_{jt'} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{70}$$

Dispatchable power and reactive power are bounded by constraints involving reserves that we formulate later.

4.6.2 Ramping limits

Real power ramping limits depend on on-off status and startup and shutdown and apply to total power including dispatchable, startup, and shutdown power:

$$p_{jt} - p^0_j \leq d_t \left(p^{\text{ru}}_j \left(u^{\text{on}}_{jt} - u^{\text{su}}_{jt}\right) + p^{\text{ru,su}}_j \left(u^{\text{su}}_{jt} + 1 - u^{\text{on}}_{jt}\right)\right) \ \forall t = t^{\text{start}}, j \in J^{\text{pr,cs}} \tag{71}$$

$$p_{jt} - p_{j,t-1} \leq d_t \left(p^{\text{ru}}_j \left(u^{\text{on}}_{jt} - u^{\text{su}}_{jt}\right) + p^{\text{ru,su}}_j \left(u^{\text{su}}_{jt} + 1 - u^{\text{on}}_{jt}\right)\right) \ \forall t > t^{\text{start}}, j \in J^{\text{pr,cs}} \tag{72}$$

$$p_{jt} - p^0_j \geq -d_t \left(p^{\text{rd}}_j u^{\text{on}}_{jt} + p^{\text{rd,sd}}_j \left(1 - u^{\text{on}}_{jt}\right)\right) \ \forall t = t^{\text{start}}, j \in J^{\text{pr,cs}} \tag{73}$$

$$p_{jt} - p_{j,t-1} \geq -d_t \left(p^{\text{rd}}_j u^{\text{on}}_{jt} + p^{\text{rd,sd}}_j \left(1 - u^{\text{on}}_{jt}\right)\right) \ \forall t > t^{\text{start}}, j \in J^{\text{pr,cs}} \tag{74}$$

The ramp up limit is enforced in the first time interval by Eq. (71) and in later intervals by Eq. (72). The ramp down limit is enforced in the first time interval by Eq. (73) and in later intervals by Eq. (74).

4.6.3 Maximum/minimum energy over multiple intervals

Producing devices and consuming devices may have limits on total energy produced or consumed over one or more consecutive time intervals. Each such constraint w is modeled as a soft constraint, with a nonnegative variable e^+_w representing the constraint violation and incurring a penalty $z_w^{\text{en,max}}$ or $z_w^{\text{en,min}}$ appearing in the objective:

$$\sum_{t \in T_w^{\text{en,max}}} d_t p_{jt} \leq e_w^{\max} + e_w^+ \;\forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \tag{75}$$

$$\sum_{t \in T_w^{\text{en,min}}} d_t p_{jt} \geq e_w^{\min} - e_w^+ \;\forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \tag{76}$$

$$e_w^+ \geq 0 \;\forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \cup W_j^{\text{en,min}} \tag{77}$$

$$z_w^{\text{en,max}} = c^{\text{e}} e_w^+ \;\forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \tag{78}$$

$$z_w^{\text{en,min}} = c^{\text{e}} e_w^+ \;\forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \tag{79}$$

4.6.4 Device reserve variable domains

Device reserve variables are nonnegative.

$$p_{jt}^{\text{rgu}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{80}$$

$$p_{jt}^{\text{rgd}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{81}$$

$$p_{jt}^{\text{scr}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{82}$$

$$p_{jt}^{\text{nsc}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{83}$$

$$p_{jt}^{\text{rru,on}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{84}$$

$$p_{jt}^{\text{rru,off}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{85}$$

$$p_{jt}^{\text{rrd,on}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{86}$$

$$p_{jt}^{\text{rrd,off}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{87}$$

$$q_{jt}^{\text{qru}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{88}$$

$$q_{jt}^{\text{qrd}} \geq 0 \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{89}$$

4.6.5 Device reserve costs

Devices may include a cost in their reserve offers. This is modeled as a cost coefficient on each device reserve provision variable.

$$z_{jt}^{\text{rgu}} = d_t c_{jt}^{\text{rgu}} p_{jt}^{\text{rgu}} \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{90}$$

$$z_{jt}^{\text{rgd}} = d_t c_{jt}^{\text{rgd}} p_{jt}^{\text{rgd}} \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{91}$$

$$z_{jt}^{\text{scr}} = d_t c_{jt}^{\text{scr}} p_{jt}^{\text{scr}} \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{92}$$

$$z_{jt}^{\text{nsc}} = d_t c_{jt}^{\text{nsc}} p_{jt}^{\text{nsc}} \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{93}$$

$$z_{jt}^{\text{rru}} = d_t \left(c_{jt}^{\text{rru,on}} p_{jt}^{\text{rru,on}} + c_{jt}^{\text{rru,off}} p_{jt}^{\text{rru,off}} \right) \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{94}$$

$$z_{jt}^{\text{rrd}} = d_t \left(c_{jt}^{\text{rrd,on}} p_{jt}^{\text{rrd,on}} + c_{jt}^{\text{rrd,off}} p_{jt}^{\text{rrd,off}} \right) \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{95}$$

$$z_{jt}^{\text{qru}} = d_t c_{jt}^{\text{qru}} q_{jt}^{\text{qru}} \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{96}$$

$$z_{jt}^{\text{qrd}} = d_t c_{jt}^{\text{qrd}} q_{jt}^{\text{qrd}} \;\forall t \in T, j \in J^{\text{pr,cs}} \tag{97}$$

4.6.6 Absolute reserve limits, based on ramp rates

A producing or consuming device may have limits on reserves, independent of real and reactive power dispatch, that are based on operating characteristics of the device. Since these constraints are independent of the dispatch, we refer to them as absolute reserve limits. The absolute reserve limits might typically be determined as an applicable ramp rate times the ramping duration for the lowest quality product considered. In general these limits are supplied as part of the reserve offer of a device and might differ from this simple formula for various reasons. Therefore we model these as parameters provided by the data. At each time scale, all the reserve products consuming ramping capability up to that time scale are included. Some products cannot be provided by some device types, and these are indicated here with bounds of 0.

$$p_{jt}^{\text{rgu}} \leq p_j^{\text{rgu,max}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{98}$$

$$p_{jt}^{\text{rgd}} \leq p_j^{\text{rgd,max}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{99}$$

$$p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} \leq p_j^{\text{scr,max}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{100}$$

$$p_{jt}^{\text{nsc}} \leq p_j^{\text{nsc,max}} \left(1 - u_{jt}^{\text{on}}\right) \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{101}$$

$$p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} + p_{jt}^{\text{rru,on}} \leq p_j^{\text{rru,on,max}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{102}$$

$$p_{jt}^{\text{nsc}} + p_{jt}^{\text{rru,off}} \leq p_j^{\text{rru,off,max}} \left(1 - u_{jt}^{\text{on}}\right) \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{103}$$

$$p_{jt}^{\text{rgd}} + p_{jt}^{\text{rrd,on}} \leq p_j^{\text{rrd,on,max}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{104}$$

$$p_{jt}^{\text{rrd,off}} \leq p_j^{\text{rrd,off,max}} \left(1 - u_{jt}^{\text{on}}\right) \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{105}$$

$$p_{jt}^{\text{rrd,off}} = 0 \ \forall t \in T, j \in J^{\text{pr}} \tag{106}$$

$$p_{jt}^{\text{nsc}} = 0 \ \forall t \in T, j \in J^{\text{cs}} \tag{107}$$

$$p_{jt}^{\text{rru,off}} = 0 \ \forall t \in T, j \in J^{\text{cs}} \tag{108}$$

4.6.7 Relative reserve limits, based on headroom to max/min, producing devices

A producing device j can provide reserves up to the space (often called headroom) between its power value p_{jt} and the appropriate operating limit p_{jt}^{max} or p_{jt}^{min}. Each reserve product occupies its own part of the appropriate headroom. Since these constraints on reserves depend on the dispatch, we refer to them as relative reserve limits. Up-reserve products consume headroom to upper operating limits, and down-reserve products consume headroom to lower operating limits. Online reserve provision variables are constrained relative to the dispatchable power p_{jt}^{on}, while offline reserve provision variables are constrained relative to the startup and shutdown power p_{jt}^{su} and p_{jt}^{sd}. Since the reserve provision variables are nonnegative, these constraints serve as the bounds on p_{jt}^{on}.

$$p_{jt}^{\text{on}} + p_{jt}^{\text{rgu}} + p_{jt}^{\text{scr}} + p_{jt}^{\text{rru,on}} \leq p_{jt}^{\text{max}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr}} \tag{109}$$

$$p_{jt}^{\text{on}} - p_{jt}^{\text{rgd}} - p_{jt}^{\text{rrd,on}} \geq p_{jt}^{\text{min}} u_{jt}^{\text{on}} \ \forall t \in T, j \in J^{\text{pr}} \tag{110}$$

$$p_{jt}^{\text{su}} + p_{jt}^{\text{sd}} + p_{jt}^{\text{nsc}} + p_{jt}^{\text{rru,off}} \leq p_{jt}^{\text{max}} \left(1 - u_{jt}^{\text{on}}\right) \ \forall t \in T, j \in J^{\text{pr}} \tag{111}$$

Reactive power reserves have similar relative limits, and the approriate upper and lower bounds are applied when the device is dispatchable or in its startup or shutdown power curve:

$$q_{jt} + q^{\text{qru}}_{jt} \leq q^{\max}_{jt} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) \; \forall t \in T, j \in J^{\text{pr}} \tag{112}$$

$$q_{jt} - q^{\text{qrd}}_{jt} \geq q^{\min}_{jt} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) \; \forall t \in T, j \in J^{\text{pr}} \tag{113}$$

Producing devices having constraints linking real and reactive power have additional constraints on reactive power reserve depending on the real power dispatch, requiring the deployment of reactive power reserves be feasible under the dispatched real power value:

$$q_{jt} + q^{\text{qru}}_{jt} \leq q^{\max,\text{p0}}_{j} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) + \beta^{\max}_{j} p_{jt} \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqmax}} \tag{114}$$

$$q_{jt} - q^{\text{qrd}}_{jt} \geq q^{\min,\text{p0}}_{j} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) + \beta^{\min}_{j} p_{jt} \quad \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqmin}} \tag{115}$$

$$q_{jt} = q^{\text{p0}}_{j} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) + \beta_{j} p_{jt} \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqe}} \tag{116}$$

$$q^{\text{qru}}_{jt} = 0 \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqe}} \tag{117}$$

$$q^{\text{qrd}}_{jt} = 0 \; \forall t \in T, j \in J^{\text{pr}} \cap J^{\text{pqe}} \tag{118}$$

4.6.8 Relative reserve limits, based on headroom to max/min, consuming devices

Relative reserve limits for consuming devices are symmetric to those for producing devices. The difference is that up-reserve products consume headroom to lower operating limits, and down-reserve products consume headroom to upper operating limits.

$$p^{\text{on}}_{jt} + p^{\text{rgd}}_{jt} + p^{\text{rrd,on}}_{jt} \leq p^{\max}_{jt} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{cs}} \tag{119}$$

$$p^{\text{on}}_{jt} - p^{\text{rgu}}_{jt} - p^{\text{scr}}_{jt} - p^{\text{rru,on}}_{jt} \geq p^{\min}_{jt} u^{\text{on}}_{jt} \; \forall t \in T, j \in J^{\text{cs}} \tag{120}$$

$$p^{\text{su}}_{jt} + p^{\text{sd}}_{jt} + p^{\text{rrd,off}}_{jt} \leq p^{\max}_{jt} \left(1 - u^{\text{on}}_{jt}\right) \; \forall t \in T, j \in J^{\text{cs}} \tag{121}$$

Relative limits on reactive power reserves, including reactive power dispatch, for consuming devices are similar to those for producing devices:

$$q_{jt} + q^{\text{qrd}}_{jt} \le q^{\max}_{jt} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) \ \forall t \in T, j \in J^{\text{cs}} \tag{122}$$

$$q_{jt} - q^{\text{qru}}_{jt} \ge q^{\min}_{jt} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) \ \forall t \in T, j \in J^{\text{cs}} \tag{123}$$

Consuming devices having constraints linking real and reactive power have additional constraints on reactive power reserve depending on the real power dispatch, requiring the deployment of reactive power reserves be feasible under the dispatched real power value:

$$q_{jt} + q^{\text{qrd}}_{jt} \le q^{\max,\text{p0}}_{j} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) + \beta^{\max}_j p_{jt} \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqmax}} \tag{124}$$

$$q_{jt} - q^{\text{qru}}_{jt} \ge q^{\min,\text{p0}}_{j} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) + \beta^{\min}_j p_{jt} \quad \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqmin}} \tag{125}$$

$$q_{jt} = q^{\text{p0}}_{j} \left(u^{\text{on}}_{jt} + \sum_{t' \in T^{\text{supc}}_{jt}} u^{\text{su}}_{jt'} + \sum_{t' \in T^{\text{sdpc}}_{jt}} u^{\text{sd}}_{jt'} \right) + \beta_j p_{jt} \ \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqe}} \tag{126}$$

$$q^{\text{qru}}_{jt} = 0 \ \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqe}} \tag{127}$$

$$q^{\text{qrd}}_{jt} = 0 \ \forall t \in T, j \in J^{\text{cs}} \cap J^{\text{pqe}} \tag{128}$$

4.6.9 Energy cost and value

Producing or consuming device real power p_{jt} results in an objective term z^{en}_{jt} that is either a cost (for producing devices) or a value (for consuming devices). The value of this energy cost (or value) term is determined by a piecewise linear convex (or concave) cost (or value) function, evaluated at the power value p_{jt}. To model this relationship, we split the power into offer (or bid) blocks and apply an objective coefficient to each block:

$$0 \le p_{jtm} \le p^{\max}_{jtm} \ \forall t \in T, j \in J^{\text{pr,cs}}, m \in M_{jt} \tag{129}$$

$$p_{jt} = \sum_{m \in M_{jt}} p_{jtm} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{130}$$

$$z^{\text{en}}_{jt} = d_t \sum_{m \in M_{jt}} c^{\text{en}}_{jtm} p_{jtm} \ \forall t \in T, j \in J^{\text{pr,cs}} \tag{131}$$

4.7 Shunt devices

Shunt real and reactive power p_{jt} and q_{jt} are:

$$p_{jt} = g^{\text{sh}}_{jt} v^2_{it} \ \forall t \in T, j \in J^{\text{sh}}, i = i_j \tag{132}$$
$$q_{jt} = -b^{\text{sh}}_{jt} v^2_{it} \ \forall t \in T, j \in J^{\text{sh}}, i = i_j \tag{133}$$

Shunt conductance and susceptance g^{sh}_{jt} and b^{sh}_{jt} are defined by a fixed step size times a variable number u^{sh}_{jt} of steps activated:

$$g^{\text{sh}}_{jt} = g^{\text{sh}}_{j} u^{\text{sh}}_{jt} \ \forall t \in T, j \in J^{\text{sh}} \tag{134}$$
$$b^{\text{sh}}_{jt} = b^{\text{sh}}_{j} u^{\text{sh}}_{jt} \ \forall t \in T, j \in J^{\text{sh}} \tag{135}$$

For each shunt, the number of steps activated is an integer that is bounded by input data:

$$u^{\text{sh}}_{jt} \in \{\dots, -1, 0, 1, \dots\} \ \forall t \in T, j \in J^{\text{sh}} \tag{136}$$
$$u^{\text{sh,min}}_{j} \le u^{\text{sh}}_{jt} \le u^{\text{sh,max}}_{j} \ \forall t \in T, j \in J^{\text{sh}} \tag{137}$$

4.8 Branch devices

4.8.1 AC branch flow limits and penalties

AC branch flows are subject to limits that are modeled as soft constraints, with overload incurring a penalty that appears in the objective. For each AC branch j and each interval t, the flow limit is applied to the apparent power at the from and to buses, the overload is represented by a nonnegative variable s^+_{jt}, and the penalty is z^{s}_{jt}:

$$0 \le s^+_{jt} \ \forall t \in T, j \in J^{\text{ac}} \tag{138}$$
$$z^{\text{s}}_{jt} = d_t c^{\text{s}} s^+_{jt} \ \forall t \in T, j \in J^{\text{ac}} \tag{139}$$
$$\left(\left(p^{\text{fr}}_{jt}\right)^2 + \left(q^{\text{fr}}_{jt}\right)^2 \right)^{1/2} \le s^{\text{max}}_{j} + s^+_{jt} \ \forall t \in T, j \in J^{\text{ac}} \tag{140}$$
$$\left(\left(p^{\text{to}}_{jt}\right)^2 + \left(q^{\text{to}}_{jt}\right)^2 \right)^{1/2} \le s^{\text{max}}_{j} + s^+_{jt} \ \forall t \in T, j \in J^{\text{ac}} \tag{141}$$

4.8.2 AC branch controls

AC branches j have variables ϕ_{jt} and τ_{jt} for the phase difference and winding ratio in intervals t. These variables are bounded by parameters from data. AC lines have phase difference equal to 0 and winding ratio equal to 1:

$$\phi_{jt} = 0 \ \forall t \in T, j \in J^{\text{ln}} \tag{142}$$
$$\tau_{jt} = 1 \ \forall t \in T, j \in J^{\text{ln}} \tag{143}$$

Fixed phase difference transformers have phase difference equal to the given initial value:

$$\phi_{jt} = \phi^0_{j} \ \forall t \in T, j \in J^{\text{fpd}} \tag{144}$$

Fixed winding ratio transformers have winding ratio equal to the given initial value:

$$\tau_{jt} = \tau_j^0 \ \forall t \in T, j \in J^{\text{fwr}} \tag{145}$$

Variable phase difference transformers have phase difference bounds:

$$\phi_j^{\min} \le \phi_{jt} \le \phi_j^{\max} \ \forall t \in T, j \in J^{\text{vpd}} \tag{146}$$

Variable winding ratio transformers have winding ratio bounds:

$$\tau_j^{\min} \le \tau_{jt} \le \tau_j^{\max} \ \forall t \in T, j \in J^{\text{vwr}} \tag{147}$$

4.8.3 AC branch flows

For each AC branch device j and each interval t, real and reactive power flows into the branch at the from and to buses are represented by variables p^{fr}, q^{fr}, p^{to}_{jt}, and q^{to}_{jt}. These flows are defined by nonlinear functions of branch parameters and controls and bus voltages, including bus voltage angles θ_{it}, and are equal to 0 if the device is offline:

$$\begin{aligned}
p^{\text{fr}}_{jt} &= u^{\text{on}}_{jt}((g^{\text{sr}}_j + g^{\text{fr}}_j)v_{it}^2/\tau_{jt}^2 + (-g^{\text{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad - b^{\text{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \ \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j && (148) \\
q^{\text{fr}}_{jt} &= u^{\text{on}}_{jt}((-b^{\text{sr}}_j - b^{\text{fr}}_j - b^{\text{ch}}_j/2)v_{it}^2/\tau_{jt}^2 + (b^{\text{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad - g^{\text{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \ \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j && (149) \\
p^{\text{to}}_{jt} &= u^{\text{on}}_{jt}((g^{\text{sr}}_j + g^{\text{to}}_j)v_{i't}^2 + (-g^{\text{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad + b^{\text{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \ \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j && (150) \\
q^{\text{to}}_{jt} &= u^{\text{on}}_{jt}((-b^{\text{sr}}_j - b^{\text{to}}_j - b^{\text{ch}}_j/2)v_{i't}^2 + (b^{\text{sr}}_j \cos(\theta_{it} - \theta_{i't} - \phi_{jt}) \\
&\quad + g^{\text{sr}}_j \sin(\theta_{it} - \theta_{i't} - \phi_{jt}))v_{it}v_{i't}/\tau_{jt}) \ \forall t \in T, j \in J^{\text{ac}}, i = i^{\text{fr}}_j, i' = i^{\text{to}}_j && (151)
\end{aligned}$$

4.8.4 DC lines

For each DC line j and each interval t, the real and reactive power flows into the line at the from and to buses are represented by variables p^{fr}_{jt}, q^{fr}_{jt}, p^{to}_{jt}, q^{to}_{jt}. Bounds on these flows are defined by input data:

$$\begin{aligned}
-p_j^{\text{dc,max}} &\le p^{\text{fr}}_{jt} \le p_j^{\text{dc,max}} \ \forall t \in T, j \in J^{\text{dc}} && (152) \\
q_j^{\text{dc,min,fr}} &\le q^{\text{fr}}_{jt} \le q_j^{\text{dc,max,fr}} \ \forall t \in T, j \in J^{\text{dc}} && (153) \\
-p_j^{\text{dc,max}} &\le p^{\text{to}}_{jt} \le p_j^{\text{dc,max}} \ \forall t \in T, j \in J^{\text{dc}} && (154) \\
q_j^{\text{dc,min,to}} &\le q^{\text{to}}_{jt} \le q_j^{\text{dc,max,to}} \ \forall t \in T, j \in J^{\text{dc}} && (155)
\end{aligned}$$

DC line real power losses are not modeled in this formulation:

$$p^{\text{fr}}_{jt} + p^{\text{to}}_{jt} = 0 \ \forall t \in T, j \in J^{\text{dc}} \tag{156}$$

4.9 Post-contingency power flow, balance, limits

The state of the grid following a contingency is modeled by a DC power flow ignoring changes to reactive power, voltage, and losses. Each contingency is characterized by the loss of a set of branches. Non-branch devices and DC lines remaining in service are assumed to maintain their real power values. System-wide real power mismatch among this subset of devices is distributed over the buses in proportion to a prescribed slack distribution. The ensuing DC real power flows of the remaining AC branches are combined with their pre-contingency reactive power flows to form an approximation of the post-contingency AC branch apparent power flows, The approximate post-contingency AC branch apparent power flows are subject to post-contingency limits, formulated as soft constraints with any violation incurring a penalty that appears in the objective.

4.9.1 Penalty on post-contingency AC branch overload

Post-contingency AC branch overload is modeled by a nonnegative variable s^{+}_{jtk}, which incurs a penalty z^{s}_{jtk} appearing in the objective:

$$s^{+}_{jtk} \geq 0 \;\forall t \in T, k \in K, j \in J^{\mathrm{ac}}_{k} \tag{157}$$

$$z^{\mathrm{s}}_{jtk} = d_t c^{\mathrm{s}} s^{+}_{jtk} \;\forall t \in T, k \in K, j \in J^{\mathrm{ac}}_{k} \tag{158}$$

4.9.2 Post-contingency AC power flow limits

Post-contingency AC branch real power flow is modeled by a variable p_{jtk}, representing an approximation of the real power flow on AC branch j, oriented from the from bus to the to bus, in interval t, under contingency k. Changes to reactive power flow caused by contingencies are not modeled, so the post-contingency apparent power flow contraints at the from and to buses are modeled using the pre-contingency reactive power flows q^{fr}_{jt} and q^{to}_{jt}, the post-contingency real power p_{jtk}, the post-contingency apparent power overload s^{+}_{jtk}, and the post-contingency flow limit $s^{\mathrm{max,ctg}}_{j}$:

$$\left((p_{jtk})^2 + \left(q^{\mathrm{fr}}_{jt}\right)^2\right)^{1/2} \leq s^{\mathrm{max,ctg}}_{j} + s^{+}_{jtk} \;\forall t \in T, k \in K, j \in J^{\mathrm{ac}}_{k} \tag{159}$$

$$\left((p_{jtk})^2 + \left(q^{\mathrm{to}}_{jt}\right)^2\right)^{1/2} \leq s^{\mathrm{max,ctg}}_{j} + s^{+}_{jtk} \;\forall t \in T, k \in K, j \in J^{\mathrm{ac}}_{k} \tag{160}$$

4.9.3 Post-contingency AC branch real power flows

Post-contingency AC branch real power flows follow a DC flow model. For this we introduce a variable θ_{itk} for the post-contingency voltage angle at bus i in interval t in contingency k. Variables for AC branch on-off and phase difference are fixed to their pre-contingency values. Then the DC flow model is

$$p_{jtk} = -b^{\mathrm{sr}}_{j} u^{\mathrm{on}}_{jt} \left(\theta_{itk} - \theta_{i'tk} - \phi_{jt}\right) \;\forall t \in T, k \in K, j \in J^{\mathrm{ac}}_{k}, i = i^{\mathrm{fr}}_{j}, i' = i^{\mathrm{to}}_{j} \tag{161}$$

4.9.4 Post-contingency real power balance

In the DC post-contingency model, the real power injections and withdrawals of non-branch devices and DC lines are fixed to their pre-contingency values, and real power balance is enforced at each bus. Since the pre-contingency real power values reflect an AC model that includes losses, and the post-contingency AC branch flows follow a lossless DC model, there may be a nonzero system-wide real power mismatch that is represented by a variable p^{sl}_t:

$$p^{\text{sl}}_t = \sum_{j \in J^{\text{pr}}} p_{jt} - \sum_{j \in J^{\text{cs}}} p_{jt} - \sum_{j \in J^{\text{sh}}} p_{jt} \ \forall t \in T \tag{162}$$

The system slack is distributed across the buses i in proportion to fixed slack distribution coefficients α_i. Then the post-contingency power balance constraints are

$$\sum_{j \in J^{\text{ac}}_k \cap J^{\text{fr}}_i} p_{jtk} - \sum_{j \in J^{\text{ac}}_k \cap J^{\text{to}}_i} p_{jtk} = \sum_{j \in J^{\text{pr}}_i} p_{jt} - \sum_{j \in J^{\text{cs}}_i} p_{jt} - \sum_{j \in J^{\text{sh}}_i} p_{jt} - \sum_{j \in J^{\text{dc}}_k \cap J^{\text{fr}}_i} p^{\text{fr}}_{jt} - \sum_{j \in J^{\text{dc}}_k \cap J^{\text{to}}_i} p^{\text{to}}_{jt} - \alpha_i p^{\text{sl}}_t \ \forall t \in T, k \in K, i \in I \tag{163}$$

5 Data Formats and Construction of Derived Data

Input (problem) and output (solution) data formats are described in [11]. The data directly provided by the input data file is referred to as elementary data. Other data required by the problem formulation are either static data, defined in this document, or derived data, computed from elementary and static data. The output data file contains the data on solution variable values that is needed to completely reconstruct and evaluate the solution.

5.1 Static Data

Static data is data whose values are provided in this document without reference to any other file and will not change except if changed in this document. The static data items and their values are:

$$\epsilon^{\text{int}} = \text{1e-8} \tag{164}$$

$$\epsilon^{\text{time}} = \text{1e-6} \tag{165}$$

$$\epsilon^{\text{constr}} = \text{1e-8} \tag{166}$$

$$\epsilon^{\text{beta}} = \text{1e-6} \tag{167}$$

$$\epsilon^{\text{susd}} = \text{1e-6} \tag{168}$$

$$d^{\text{unit}} = \text{5e-3} \tag{169}$$

5.2 Elementary Data

Data read directly from the input data file is referred to as elementary data. Elementary data items are listed below:

- Index sets: F, I, J, K, M, N, T, W
- Subsets: J^{dc}, J^{ln}, J^{xf}, J^{pr}, J^{cs}, J^{sh}, J^{pqe}, J^{pqmax}, J^{pqmin}, F_j, J_k^{out}, M_{jt}, N^{p}, N^{q}, N_i, $W_j^{\mathrm{en,max}}$, $W_j^{\mathrm{en,min}}$, $W_j^{\mathrm{su,max}}$
- Special elements of sets: i_j, i_j^{fr}, i_j^{to}
- Parameters: $a_w^{\mathrm{en,max,start}}$, $a_w^{\mathrm{en,max,end}}$, $a_w^{\mathrm{en,min,start}}$, $a_w^{\mathrm{en,min,end}}$, $a_w^{\mathrm{su,max,start}}$, $a_w^{\mathrm{su,max,end}}$, b_j^{ch}, b_j^{fr}, b_j^{sh}, b_j^{to}, c^{e}, c_{jtm}^{en}, c^{p}, c^{q}, c_{jt}^{rgd}, c_{jt}^{rgu}, c_{jt}^{scr}, c_{jt}^{nsc}, $c_{jt}^{\mathrm{rru,off}}$, $c_{jt}^{\mathrm{rru,on}}$, $c_{jt}^{\mathrm{rrd,off}}$, $c_{jt}^{\mathrm{rrd,on}}$, c_{jt}^{qru}, c_{jt}^{qrd}, c_n^{rgu}, c_n^{rgd}, c_n^{scr}, c_n^{nsc}, c_n^{rru}, c_n^{rrd}, c_n^{qru}, c_n^{qrd}, c^{s}, c_j^{on}, c_j^{su}, c_{jf}^{sus}, c_j^{sd}, d_t, $d_{jf}^{\mathrm{dn,max}}$, $d_j^{\mathrm{dn,min}}$, $d_j^{\mathrm{dn,0}}$, $d_j^{\mathrm{up,min}}$, $d_j^{\mathrm{up,0}}$, e_w^{max}, e_w^{min}, g_j^{fr}, g_j^{sh}, g_j^{to}, $p_j^{\mathrm{dc,max}}$, p_{jtm}^{max}, p_{jt}^{max}, p_{jt}^{min}, p_j^{rd}, $p_j^{\mathrm{rd,sd}}$, p_j^{ru}, $p_j^{\mathrm{ru,su}}$, p_j^0, $p_j^{\mathrm{rgu,max}}$, $p_j^{\mathrm{rgd,max}}$, $p_j^{\mathrm{scr,max}}$, $p_j^{\mathrm{nsc,max}}$, $p_j^{\mathrm{rru,on,max}}$, $p_j^{\mathrm{rru,off,max}}$, $p_j^{\mathrm{rrd,on,max}}$, $p_j^{\mathrm{rrd,off,max}}$, $p_{nt}^{\mathrm{rru,min}}$, $p_{nt}^{\mathrm{rrd,min}}$, q_j^0, q_j^{p0}, $q_j^{\mathrm{max,p0}}$, $q_j^{\mathrm{min,p0}}$, q_{jt}^{max}, q_{jt}^{min}, $q_j^{\mathrm{dc,fr,max}}$, $q_j^{\mathrm{dc,to,max}}$, $q_j^{\mathrm{dc,fr,min}}$, $q_j^{\mathrm{dc,to,min}}$, $q_{nt}^{\mathrm{qru,min}}$, $q_{nt}^{\mathrm{qrd,min}}$, r_j^{sr}, s_j^{max}, $s_j^{\mathrm{max,ctg}}$, $u_{jt}^{\mathrm{on,max}}$, $u_{jt}^{\mathrm{on,min}}$, $u_j^{\mathrm{on,0}}$, $u_w^{\mathrm{su,max}}$, $u_j^{\mathrm{sh,0}}$, $u_j^{\mathrm{sh,max}}$, $u_j^{\mathrm{sh,min}}$, v_i^0, v_i^{max}, v_i^{min}, x_j^{sr}, β_j, β_j^{max}, β_j^{min}, σ_n^{rgd}, σ_n^{rgu}, σ_n^{scr}, σ_n^{nsc}, τ_j^0, τ_j^{max}, τ_j^{min}, θ_i^0, ϕ_j^0, ϕ_j^{max}, ϕ_j^{min}

5.3 Construction of Derived Data from Static Data and Elementary Data

Data constructed from elementary data is referred to as derived data. This section shows how the derived data is constructed from the elementary data.

The set J_k^{out} of devices outaged in contingency k contains a single element, and this element is identified as j_k^{out}. The set T of time intervals is ordered, and the first and last elements of this set are identified as t^{end} and t^{start}.

The set of buses in each reserve zone is constructed from the reserve zones for each bus:

$$I_n = \{i \in I : n \in N_i\} \ \forall n \in N \tag{170}$$

Certain union sets are constructed:

$$J^{\mathrm{ac}} = J^{\mathrm{ln}} \cup J^{\mathrm{xf}} \tag{171}$$
$$J^{\mathrm{br}} = J^{\mathrm{ac}} \cup J^{\mathrm{dc}} \tag{172}$$
$$J^{\mathrm{pr,cs}} = J^{\mathrm{pr}} \cup J^{\mathrm{cs}} \tag{173}$$
$$J^{\mathrm{pr,cs,ac}} = J^{\mathrm{pr}} \cup J^{\mathrm{cs}} \cup J^{\mathrm{ac}} \tag{174}$$

Transformers are categorized as fixed or variable with respect to phase difference and winding ratio according to the presented bounds on controls:

$$J^{\mathrm{vpd}} = \{j \in J^{\mathrm{xf}} : \phi^{\mathrm{min}} < \phi^{\mathrm{max}}\} \tag{175}$$
$$J^{\mathrm{vwr}} = \{j \in J^{\mathrm{xf}} : \tau^{\mathrm{min}} < \tau^{\mathrm{max}}\} \tag{176}$$
$$J^{\mathrm{fpd}} = J^{\mathrm{xf}} \setminus J^{\mathrm{vpd}} \tag{177}$$
$$J^{\mathrm{fwr}} = J^{\mathrm{xf}} \setminus J^{\mathrm{vwr}} \tag{178}$$

Devices at each bus:

$$J_i^{\text{fr}} = \{j \in J^{\text{br}} : i_j^{\text{fr}} = i\} \ \forall i \in I \tag{179}$$
$$J_i^{\text{to}} = \{j \in J^{\text{br}} : i_j^{\text{to}} = i\} \ \forall i \in I \tag{180}$$
$$J_i^{\text{br}} = J_i^{\text{fr}} \cup J_i^{\text{to}} \ \forall i \in I \tag{181}$$
$$J_i^{\text{pr}} = \{j \in J^{\text{pr}} : i_j = i\} \ \forall i \in I \tag{182}$$
$$J_i^{\text{cs}} = \{j \in J^{\text{cs}} : i_j = i\} \ \forall i \in I \tag{183}$$
$$J_i^{\text{sh}} = \{j \in J^{\text{sh}} : i_j = i\} \ \forall i \in I \tag{184}$$
$$J_i = J_i^{\text{br}} \cup J_i^{\text{pr}} \cup J_i^{\text{cs}} \cup J_i^{\text{sh}} \ \forall i \in I \tag{185}$$

Devices in each reserve zone:

$$J_n^{\text{pr}} = \cup_{i \in I_n} J_i^{\text{pr}} \ \forall n \in N \tag{186}$$
$$J_n^{\text{cs}} = \cup_{i \in I_n} J_i^{\text{cs}} \ \forall n \in N \tag{187}$$
$$J_n^{\text{pr,cs}} = J_n^{\text{pr}} \cup J_n^{\text{cs}} \ \forall n \in N \tag{188}$$

A device is outaged in a given time interval if it is prevented from operating by either its on-off state bounds or its minimum downtime and prior downtime at the start of the model horizon:

$$T_j^{\text{out}} = \{t \in T : u_{jt}^{\text{on,max}} = 0\} \ \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{up},0} > 0 \tag{189}$$
$$T_j^{\text{out}} = \{t \in T : u_{jt}^{\text{on,max}} = 0\} \cup \{t \in T : d_j^{\text{dn},0} + a_t^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{dn,min}}\} \quad \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{dn},0} > 0 \tag{190}$$

Note that Eqs. (189) and (190) define the symbol T_j^{out} under two different conditions on $j \in J^{\text{pr,cs}}$, and every $j \in J^{\text{pr,cs}}$ is covered by exactly one of these conditions (see Eqs. (242) to (245)). Eq. (189) treats the case $d_j^{\text{up},0} > 0$, and Eq. (190) treats the case $d_j^{\text{dn},0} > 0$. Note also the use of ϵ^{time} in Eq. (190). This ensures that the two sides of the unmodified strict inequality $d_j^{\text{dn},0} + a_t^{\text{start}} < d_j^{\text{dn,min}}$ are deemed equal if they are very close in value, as could occur in floating point arithmetic. In general in the construction of derived data, when two time values need to be compared by a strict inequality in order to construct a data item, we use ϵ^{time} in this way. The elementary data is constructed in such a way as to ensure that each time duration specified in this document is an integer multiple of a prescribed minimum time unit d^{unit} (see Eqs. (295) to (306)). Therefore, in time comparisons used in constructing the derived data, the two compared quantities will either be very close (i.e. within ϵ^{time}) or far apart (i.e. not within $d^{\text{unit}}/2$), so the comparisons will be unambiguous.

Similarly, a device is must-run in a given interval if it is required to be operating by either its on-off state bounds or its minimum uptime and prior uptime at the start of the model horizon:

$$T_j^{\text{mr}} = \{t \in T : u_{jt}^{\text{on,min}} = 1\} \ \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{dn},0} > 0 \tag{191}$$
$$T_j^{\text{mr}} = \{t \in T : u_{jt}^{\text{on,min}} = 1\} \cup \{t \in T : d_j^{\text{up},0} + a_t^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{up,min}}\} \quad \forall j \in J^{\text{pr,cs}} \text{ with } d_j^{\text{up},0} > 0 \tag{192}$$

The devices in service in a given contingency are those that are not out of service:

$$J_k = J \setminus J_k^{\text{out}} \; \forall k \in K \tag{193}$$

Certain useful intersection sets are constructed:

$$J_k^{\text{ac}} = J^{\text{ac}} \cap J_k \; \forall k \in K \tag{194}$$
$$J_k^{\text{dc}} = J^{\text{dc}} \cap J_k \; \forall k \in K \tag{195}$$
$$J_k^{\text{br}} = J^{\text{br}} \cap J_k \; \forall k \in K \tag{196}$$

Conductance and susceptance of the series element of each AC branch is constructed:

$$g_j^{\text{sr}} = r_j^{\text{sr}}/((r^{\text{sr}})^2 + (x^{\text{sr}})^2) \; \forall j \in J^{\text{ac}} \tag{197}$$
$$b_j^{\text{sr}} = -x_j^{\text{sr}}/((r^{\text{sr}})^2 + (x^{\text{sr}})^2) \; \forall j \in J^{\text{ac}} \tag{198}$$

Time interval start, end, and midpoint times are constructed from interval durations:

$$a_t^{\text{start}} = \sum_{t'<t} d_{t'} \; \forall t \in T \tag{199}$$
$$a_t^{\text{end}} = \sum_{t'\leq t} d_{t'} \; \forall t \in T \tag{200}$$
$$a_t^{\text{mid}} = (a_t^{\text{start}} + a_t^{\text{end}})/2 \; \forall t \in T \tag{201}$$

Sets of time intervals for minimum downtime and minimum uptime constraints are constructed by:

$$T_{jt}^{\text{dn,min}} = \{t' < t : a_t^{\text{start}} - a_{t'}^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{dn,min}}\} \; \forall t \in T, j \in J^{\text{pr,cs}} \tag{202}$$
$$T_{jt}^{\text{up,min}} = \{t' < t : a_t^{\text{start}} - a_{t'}^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{up,min}}\} \; \forall t \in T, j \in J^{\text{pr,cs}} \tag{203}$$

Startup and shutdown power curves are derived from startup and shutdown ramp rates:

$$p_{jtt'}^{\text{supc}} = p_{jt'}^{\text{min}} - p_j^{\text{ru,su}}(a_{t'}^{\text{end}} - a_t^{\text{end}}) \; \forall j \in J^{\text{pr,cs}}, t < t' \in T \tag{204}$$
$$T_{jt}^{\text{supc}} = \{t' > t : p_{jtt'}^{\text{supc}} > 0\} \; \forall j \in J^{\text{pr,cs}}, t \in T \tag{205}$$
$$p_{jtt'}^{\text{sdpc}} = p_{j,t'-1}^{\text{min}} - p_j^{\text{rd,sd}}(a_t^{\text{end}} - a_{t'}^{\text{start}}) \; \forall j \in J^{\text{pr,cs}}, t, t' \in T, t \geq t' > t^{\text{start}} \tag{206}$$
$$p_{jtt'}^{\text{sdpc}} = p_j^0 - p_j^{\text{rd,sd}}(a_t^{\text{end}} - a_{t'}^{\text{start}}) \; \forall j \in J^{\text{pr,cs}}, t, t' \in T, t \geq t' = t^{\text{start}} \tag{207}$$
$$T_{jt}^{\text{sdpc}} = \{t' \leq t : p_{jtt'}^{\text{sdpc}} > 0\} \; \forall j \in J^{\text{pr,cs}}, t \in T \tag{208}$$

The elementary data is constructed in such a way as to ensure that the startup and shutdown power curve values $p_{jtt'}^{\text{supc}}$ and $p_{jtt'}^{\text{sdpc}}$ are not close to zero (see Eqs. (313) and (314)). Therefore the definitions of the startup and shutdown time interval sets T_{jt}^{supc} and T_{jt}^{sdpc} are unambiguous in floating point arithmetic.

Sets of time intervals for modeling downtime-dependent startup costs are derived from elementary data on maximum prior downtime for each startup state:

$$T_{jft}^{\text{sus}} = \{t' \in T : t' < t, a_t^{\text{start}} - a_{t'}^{\text{start}} \leq d_{jf}^{\text{dn,max}} + \epsilon^{\text{time}}\} \; \forall j \in J^{\text{pr,cs}}, f \in F_j, t \in T \tag{209}$$
$$T_{jf}^{\text{sus}} = \{t \in T : d_j^{\text{dn,0}} + a_t^{\text{start}} > d_{jf}^{\text{dn,max}} + \epsilon^{\text{time}}\} \; \forall j \in J^{\text{pr,cs}}, f \in F_j \tag{210}$$

Sets of time intervals for constraints modeling maximum startups in a prescribed interval are constructed by:

$$T_w^{\text{su,max}} = \{t \in T : a_w^{\text{su,max,start}} \le a_t^{\text{start}} + \epsilon^{\text{time}} \text{ and } a_t^{\text{start}} + \epsilon^{\text{time}} < a_w^{\text{su,max,end}}\} \quad \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{su,max}} \tag{211}$$

Sets of time intervals for constraints modeling maximum or minimum energy over a prescribed interval are constructed by:

$$T_w^{\text{en,max}} = \{t \in T : a_w^{\text{en,max,start}} + \epsilon^{\text{time}} < a_t^{\text{mid}} \text{ and } a_t^{\text{mid}} \le a_w^{\text{en,max,end}} + \epsilon^{\text{time}}\} \quad \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \tag{212}$$

$$T_w^{\text{en,min}} = \{t \in T : a_w^{\text{en,min,start}} + \epsilon^{\text{time}} < a_t^{\text{mid}} \text{ and } a_t^{\text{mid}} \le a_w^{\text{en,min,end}} + \epsilon^{\text{time}}\} \quad \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \tag{213}$$

Bus participation factors for the post-contingency power flow and balance model are uniform and set up to sum to 1:

$$\alpha_i = 1/|I| \ \forall i \in I \tag{214}$$

5.4 Output data

The output data file needs to contain the following output data items, so as to completely reconstruct the solution:

- Bus voltage magnitudes and angles: v_{it}, θ_{it} for $t \in T$, $i \in I$.
- Shunt steps activated: u_{jt}^{sh} for $t \in T$, $j \in J^{\text{sh}}$.
- AC line on-off status: u_{jt}^{on} for $t \in T$, $j \in J^{\text{ln}}$.
- Transformer on-off status and control variables: u_{jt}^{on}, ϕ_{jt}, τ_{jt} for $t \in T$, $j \in J^{\text{xf}}$.
- DC line real and reactive power flows: p_{jt}^{fr}, q_{jt}^{fr}, q_{jt}^{to} for $t \in T$, $j \in J^{\text{dc}}$.
- Producing and consuming device commitment, power dispatch, and reserve provision: u_{jt}^{on}, p_{jt}^{on}, q_{jt}, p_{jt}^{rgu}, p_{jt}^{rgd}, p_{jt}^{scr}, p_{jt}^{nsc}, $p_{jt}^{\text{rru,on}}$, $p_{jt}^{\text{rru,off}}$, $p_{jt}^{\text{rrd,on}}$, $p_{jt}^{\text{rrd,off}}$, q_{jt}^{qru}, q_{jt}^{qrd} for $t \in T$, $j \in J^{\text{pr,cs}}$.

6 Data Properties

This section describes properties that the input data of the problem should have. For an instance of the problem, these properties can be checked, and any violation should be treated as a data error. Python code checking these properties is available in [10], which relies on a data model code in [6]. We note here that many of these properties concern the mutual consistency of sets of parameters defining lower and upper bounds on variables or expressions. In such cases we typically assert only that the lower bound is less than or equal to the upper bound. In particular, the two bounds may be equal unless this is explicitly ruled out. The performance of some solution techniques may be affected by variables whose lower and upper bounds are equal.

The index sets F, I, J, K, M, N, T, W, are viewed as pairwise disjoint, even if some of the strings or integers representing individual elements in some of these may be the same.

The subsets $W_j^{\text{en,max}}$, $W_j^{\text{en,min}}$, and $W^{\text{su,max}}$ of W, over $j \in J^{\text{pr,cs}}$ are pairwise disjoint.

Devices include branch devices and non-branch devices:

$$J^{\text{br}} \subset J \tag{215}$$

Branch devices include AC branches and DC lines:

$$J^{\text{br}} = J^{\text{ac}} \cup J^{\text{dc}} \tag{216}$$

$$J^{\text{ac}} \cap J^{\text{dc}} = \{\} \tag{217}$$

AC branches include AC lines and transformers:

$$J^{\text{ac}} = J^{\text{ln}} \cup J^{\text{xf}} \tag{218}$$

$$J^{\text{ln}} \cap J^{\text{xf}} = \{\} \tag{219}$$

Transformers have either fixed or variable phase difference:

$$J^{\text{xf}} = J^{\text{fpd}} \cup J^{\text{vpd}} \tag{220}$$

$$J^{\text{fpd}} \cap J^{\text{vpd}} = \{\} \tag{221}$$

Transformers have either fixed or variable winding ratio:

$$J^{\text{xf}} = J^{\text{fwr}} \cup J^{\text{vwr}} \tag{222}$$

$$J^{\text{fwr}} \cap J^{\text{vwr}} = \{\} \tag{223}$$

No transformer is both variable phase difference and variable winding ratio:

$$J^{\text{vpd}} \cap J^{\text{vwr}} = \{\} \tag{224}$$

Non-branch devices include shunts, producing devices, and consuming devices:

$$J \setminus J^{\text{br}} = J^{\text{sh}} \cup J^{\text{pr}} \cup J^{\text{cs}} \tag{225}$$

$$J^{\text{sh}} \cap J^{\text{pr}} = \{\} \tag{226}$$

$$J^{\text{sh}} \cap J^{\text{cs}} = \{\} \tag{227}$$

$$J^{\text{pr}} \cap J^{\text{cs}} = \{\} \tag{228}$$

The data format implies

$$J^{\text{pqmin}} = J^{\text{pqmax}} \tag{229}$$

This is not a necessary property for this formulation, but it can be assumed to hold, given the data format.

The devices outaged in a contingency are branch devices:

$$J_k^{\text{out}} \subset J^{\text{br}} \; \forall k \in K \tag{230}$$

Each contingency outages exactly one device:

$$J_k^{\text{out}} = \{j_k^{\text{out}}\} \; \forall k \in K \tag{231}$$

In the base case and in each contingency, the graph consisting of all buses and all AC branches in service in the prior operating point is connected:

$$\text{The bus-branch graph on } (I, \{j \in J^{\text{ac}} : u_t^{\text{on},0} = 1\}) \text{ is connected} \tag{232}$$

$$\text{The bus-branch graph on } (I, \{j \in J_k^{\text{ac}} : u_t^{\text{on},0} = 1\}) \text{ is connected } \forall k \in K \tag{233}$$

For each device, the prior on-off status is a binary integer:

$$u_j^{\text{on},0} \in \{0, 1\} \; \forall j \in J^{\text{pr,cs,ac}} \tag{234}$$

For each device, on-off status bounds are binary integers and are mutually consistent:

$$u_{jt}^{\max} \in \{0, 1\} \; \forall t \in T, j \in J^{\text{pr,cs}} \tag{235}$$

$$u_{jt}^{\min} \in \{0, 1\} \; \forall t \in T, j \in J^{\text{pr,cs}} \tag{236}$$

$$u_{jt}^{\min} \leq u_{jt}^{\max} \; \forall t \in T, j \in J^{\text{pr,cs}} \tag{237}$$

For each shunt, the bounds on the number of activated steps and the initial number of activated steps are nonnegative integers and are mutually consistent:

$$u_j^{\text{sh,max}} \in \{0, 1, \dots\} \; \forall j \in J^{\text{sh}} \tag{238}$$

$$u_j^{\text{sh,min}} \in \{0, 1, \dots\} \; \forall j \in J^{\text{sh}} \tag{239}$$

$$u_j^{\text{sh},0} \in \{0, 1, \dots\} \; \forall j \in J^{\text{sh}} \tag{240}$$

$$u_j^{\text{sh,min}} \leq u^{\text{sh},0} \leq u_j^{\text{sh,max}} \; \forall j \in J^{\text{sh}} \tag{241}$$

For each device, the prior uptime and prior downtime are both nonnegative, and exactly one of them is positive:

$$d_j^{\text{up},0} \geq 0 \; \forall j \in J \tag{242}$$

$$d_j^{\text{dn},0} \geq 0 \; \forall j \in J \tag{243}$$

$$d_j^{\text{up},0} = 0 \text{ or } d_j^{\text{dn},0} = 0 \; \forall j \in J \tag{244}$$

$$d_j^{\text{up},0} > 0 \text{ or } d_j^{\text{dn},0} > 0 \; \forall j \in J \tag{245}$$

For each bus, the prior voltage value and the voltage bounds are positive and mutually consistent:

$$0 < v_i^{\min} \leq v_i^0 \leq v_i^{\max} \; \forall i \in I \tag{246}$$

For each AC branch, the impedance of the series element is non-zero:

$$r_j^{\mathrm{sr}} \neq 0 \text{ or } x_j^{\mathrm{sr}} \neq 0 \ \forall j \in J^{\mathrm{ac}} \tag{247}$$

For each transformer, the bounds and initial values on phase difference and winding ratio are mutually consistent, and the winding ratios are positive:

$$\phi_j^{\min} \leq \phi_j^0 \leq \phi_j^{\max} \ \forall j \in J^{\mathrm{xf}} \tag{248}$$
$$0 < \tau_j^{\min} \leq \tau_j^0 \leq \tau_j^{\max} \ \forall j \in J^{\mathrm{xf}} \tag{249}$$

For each transformer, either the bounds on phase difference are equal to each other, or the bounds on winding ratio are equal to each other:

$$\tau_j^{\min} = \tau_j^{\max} \text{ or } \phi_j^{\min} = \phi_j^{\max} \ \forall j \in J^{\mathrm{xf}} \tag{250}$$

For each DC line, the bounds on reactive power at each bus are mutually consistent, the bounds on real power are nonnegative, and the bounds permit 0 flow:

$$p_j^{\mathrm{dc,max}} \geq 0 \ \forall j \in J^{\mathrm{dc}} \tag{251}$$
$$q_j^{\mathrm{dc,fr,min}} \leq 0 \leq q_j^{\mathrm{dc,fr,max}} \ \forall j \in J^{\mathrm{dc}} \tag{252}$$
$$q_j^{\mathrm{dc,to,min}} \leq 0 \leq q_j^{\mathrm{dc,to,max}} \ \forall j \in J^{\mathrm{dc}} \tag{253}$$

For each producing or consuming device, the p bounds are nonnegative, and p and q bounds are mutually consistent:

$$0 \leq p_{jt}^{\min} \leq p_{jt}^{\max} \ \forall t \in T, j \in J^{\mathrm{pr,cs}} \tag{254}$$
$$q_{jt}^{\min} \leq q_{jt}^{\max} \ \forall t \in T, j \in J^{\mathrm{pr,cs}} \tag{255}$$

For each producing or consuming device, the ramping limits and the absolute bounds on reserve provision are nonnegative:

$$p_j^{\mathrm{ru}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{256}$$
$$p_j^{\mathrm{rd}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{257}$$
$$p_j^{\mathrm{ru,su}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{258}$$
$$p_j^{\mathrm{rd,sd}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{259}$$
$$p_j^{\mathrm{rgu,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{260}$$
$$p_j^{\mathrm{rgd,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{261}$$
$$p_j^{\mathrm{scr,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{262}$$
$$p_j^{\mathrm{nsc,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{263}$$
$$p_j^{\mathrm{rru,on,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{264}$$
$$p_j^{\mathrm{rru,off,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{265}$$
$$p_j^{\mathrm{rrd,on,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{266}$$
$$p_j^{\mathrm{rrd,off,max}} \geq 0 \ \forall j \in J^{\mathrm{pr,cs}} \tag{267}$$

For each producing or consuming device, the bid or offer blocks have nonnegative width:

$$p^{\max}_{jtm} \geq 0 \; \forall t \in T, j \in J^{\text{pr,cs}}, m \in M_{jt} \tag{268}$$

For each producing or consuming device with p-q constraints, the bounds are mutually consistent:

$$q^{\min,\text{p0}}_j \leq q^{\max,\text{p0}}_j \; \forall j \in J^{\text{pqmax}} \cap J^{\text{pqmin}} \tag{269}$$

For each AC branch device, the apparent power flow limits are positive, and the contingency limits are not tighter than the base case limits:

$$s^{\max}_j > 0 \; \forall j \in J^{\text{ac}} \tag{270}$$

$$s^{\max,\text{ctg}}_j \geq s^{\max}_j \; \forall j \in J^{\text{ac}} \tag{271}$$

For each time interval, the duration is positive:

$$d_t > 0 \; \forall t \in T \tag{272}$$

The reserve requirements are nonnegative:

$$\sigma^{\text{rgu}}_n \geq 0 \; \forall n \in N^{\text{p}} \tag{273}$$

$$\sigma^{\text{rgd}}_n \geq 0 \; \forall n \in N^{\text{p}} \tag{274}$$

$$\sigma^{\text{scr}}_n \geq 0 \; \forall n \in N^{\text{p}} \tag{275}$$

$$\sigma^{\text{nsc}}_n \geq 0 \; \forall n \in N^{\text{p}} \tag{276}$$

$$p^{\text{rru,min}}_{nt} \geq 0 \; \forall t \in T, n \in N^{\text{p}} \tag{277}$$

$$p^{\text{rrd,min}}_{nt} \geq 0 \; \forall t \in T, n \in N^{\text{p}} \tag{278}$$

$$q^{\text{qru,min}}_{nt} \geq 0 \; \forall t \in T, n \in N^{\text{q}} \tag{279}$$

$$q^{\text{qrd,min}}_{nt} \geq 0 \; \forall t \in T, n \in N^{\text{q}} \tag{280}$$

Constraint relaxation penalties are strictly positive:

$$c^{\text{e}} > 0 \tag{281}$$

$$c^{\text{p}} > 0 \tag{282}$$

$$c^{\text{q}} > 0 \tag{283}$$

$$c^{\text{s}} > 0 \tag{284}$$

$$c^{\text{rgu}}_n > 0 \; \forall n \in N^{\text{p}} \tag{285}$$

$$c^{\text{rgd}}_n > 0 \; \forall n \in N^{\text{p}} \tag{286}$$

$$c^{\text{scr}}_n > 0 \; \forall n \in N^{\text{p}} \tag{287}$$

$$c^{\text{nsc}}_n > 0 \; \forall n \in N^{\text{p}} \tag{288}$$

$$c^{\text{rru}}_n > 0 \; \forall n \in N^{\text{p}} \tag{289}$$

$$c^{\text{rrd}}_n > 0 \; \forall n \in N^{\text{p}} \tag{290}$$

$$c^{\text{qru}}_n > 0 \; \forall n \in N^{\text{q}} \tag{291}$$

$$c^{\text{qrd}}_n > 0 \; \forall n \in N^{\text{q}} \tag{292}$$

Certain index sets are nonempty:

$$I \neq \{\} \tag{293}$$
$$T \neq \{\} \tag{294}$$

All elementary and derived time duration parameters are integer multiples of the minimum time unit d^{unit}:

$$|d_t/(2d^{\text{unit}}) - u| < \epsilon^{\text{int}} \text{ for some integer } u \tag{295}$$
$$|d_j^{\text{up},0}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}} \tag{296}$$
$$|d_j^{\text{dn},0}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}} \tag{297}$$
$$|d_j^{\text{up,min}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}} \tag{298}$$
$$|d_j^{\text{dn,min}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}} \tag{299}$$
$$|d_{jf}^{\text{dn,max}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ f \in F_j \tag{300}$$
$$|a_w^{\text{en,max,start}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ w \in W_j^{\text{en,max}} \tag{301}$$
$$|a_w^{\text{en,max,end}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ w \in W_j^{\text{en,max}} \tag{302}$$
$$|a_w^{\text{en,min,start}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ w \in W_j^{\text{en,min}} \tag{303}$$
$$|a_w^{\text{en,min,end}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ w \in W_j^{\text{en,min}} \tag{304}$$
$$|a_w^{\text{su,max,start}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ w \in W_j^{\text{su,max}} \tag{305}$$
$$|a_w^{\text{su,max,end}}/d^{\text{unit}} - u| < \epsilon^{\text{int}} \text{ for some integer } u, \text{ for all } j \in J^{\text{pr,cs}},\ w \in W_j^{\text{su,max}} \tag{306}$$

The end times of all multi-interval constraints do not exceed the model horizon end time:

$$a_w^{\text{en,max,end}} \leq \sum_{t \in T} d_t + \epsilon^{\text{time}}\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \tag{307}$$
$$a_w^{\text{en,min,end}} \leq \sum_{t \in T} d_t + \epsilon^{\text{time}}\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \tag{308}$$
$$a_w^{\text{su,max,end}} \leq \sum_{t \in T} d_t + \epsilon^{\text{time}}\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{su,max}} \tag{309}$$

The start times of all multi-interval constraints do not exceed the corresponding end times:

$$a_w^{\text{en,max,start}} \leq a_w^{\text{en,max,end}} + \epsilon^{\text{time}}\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,max}} \tag{310}$$
$$a_w^{\text{en,min,start}} \leq a_w^{\text{en,min,end}} + \epsilon^{\text{time}}\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{en,min}} \tag{311}$$
$$a_w^{\text{su,max,start}} \leq a_w^{\text{su,max,end}} + \epsilon^{\text{time}}\ \forall j \in J^{\text{pr,cs}}, w \in W_j^{\text{su,max}} \tag{312}$$

Startup and shutdown trajectory p-values are not too close to 0:

$$|p_{jtt'}^{\text{supc}}| \geq \epsilon^{\text{susd}}\ \forall j \in J^{\text{pr,cs}}, t < t' \in T \tag{313}$$
$$|p_{jtt'}^{\text{sdpc}}| \geq \epsilon^{\text{susd}}\ \forall j \in J^{\text{pr,cs}}, t \geq t' \in T \tag{314}$$

Linear coefficients in constraints linking device real and reactive power are not too close to 0 or to each other, unless they are exactly 0 or equal to each other:

$$|\beta_j| \geq \epsilon^{\text{beta}} \text{ or } \beta_j = 0 \ \forall j \in J^{\text{pqe}} \tag{315}$$
$$|\beta_j^{\max}| \geq \epsilon^{\text{beta}} \text{ or } \beta_j^{\max} = 0 \ \forall j \in J^{\text{pqmax}} \tag{316}$$
$$|\beta_j^{\min}| \geq \epsilon^{\text{beta}} \text{ or } \beta_j^{\min} = 0 \ \forall j \in J^{\text{pqmin}} \tag{317}$$
$$|\beta_j^{\max} - \beta_j^{\min}| \geq \epsilon^{\text{beta}} \text{ or } \beta_j^{\max} = \beta_j^{\min} \ \forall j \in J^{\text{pqmax}} \cap J^{\text{pqmin}} \tag{318}$$

Device energy cost/value functions cover the energy upper bounds and startup and shutdown trajectories:

$$\sum_{m \in M_{jt}} p_{jtm}^{\max} \geq p_{jt}^{\max} \ \forall j \in J^{\text{pr,cs}}, t \in T \tag{319}$$
$$\sum_{m \in M_{jt}} p_{jtm}^{\max} \geq p_{jtt'}^{\text{supc}} \ \forall j \in J^{\text{pr,cs}}, t \in T, t' \in T_{jt}^{\text{supc}} \tag{320}$$
$$\sum_{m \in M_{jt}} p_{jtm}^{\max} \geq p_{jtt'}^{\text{sdpc}} \ \forall j \in J^{\text{pr,cs}}, t \in T, t' \in T_{jt}^{\text{sdpc}} \tag{321}$$

For each producing or consuming device, each shutdown interval or shutdown trajectory does not intersect with the next subsequent startup interval or startup trajectory that is feasible with respect to minimum downtime:

$$a_{t''}^{\text{start}} - a_{t'}^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{min,dn}} \ \forall j \in J^{\text{pr,cs}}, t \in T, t' \in T_{jt}^{\text{sdpc}}, t'' \in T_{jt}^{\text{supc}} \tag{322}$$
$$a_{t}^{\text{start}} - a_{t'}^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{min,dn}} \ \forall j \in J^{\text{pr,cs}}, t \in T, t' \in T_{jt}^{\text{sdpc}} \tag{323}$$
$$a_{t''}^{\text{start}} - a_{t}^{\text{start}} + \epsilon^{\text{time}} < d_j^{\text{min,dn}} \ \forall j \in J^{\text{pr,cs}}, t \in T, t > t^{\text{start}}, t'' \in T_{j,t-1}^{\text{supc}} \tag{324}$$

These properties ensure that for any $j \in J^{\text{pr,cs}}$ and any $t \in T$, at most one of the three terms p_{jt}^{on}, p_{jt}^{su}, and p_{jt}^{sd} can be nonzero.

For each producing or consuming device, the must run and planned outage statuses derived from initial uptime and downtime and minimum uptime and downtime are consistent with upper and lower bounds on the commitment statuses:

$$T_j^{\text{mr}} \cap T_j^{\text{out}} = \{\} \ \forall j \in J^{\text{pr,cs}} \tag{325}$$

For every reactive power zone and every time interval, the total reactive power reserve capability over all contributing devices does not fall short of the total reactive power reserve requirement:

$$\sum_{j \in J_n^{\text{pr,cs}}} \left(q_{jt}^{\max} - q_{jt}^{\min} \right) \geq q_{nt}^{\text{qru,min}} + q_{nt}^{\text{qrd,min}} + \epsilon^{\text{constr}} \ \forall n \in N^{\text{q}}, t \in T \tag{326}$$

For each producing or consuming device, the upper and lower bounds on commitment status variables, the minimum uptime and downtime constraints, the multi-interval maximum startups constraints, and the uptime and downtime in the prior operating point are mutually consistent:

$$\text{There exist } u_{jt}^{\text{on}}, u_{jt}^{\text{su}}, u_{jt}^{\text{sd}}, \text{ for } t \in T, \text{ satisfying Eqs. (48) to (58), for every } j \in J^{\text{pr,cs}} \tag{327}$$

Therefore we may define for each producing or consuming device $j \in J^{\mathrm{pr,cs}}$ a *prior operating point (POP) commitment solution* $u^{\mathrm{on,pop}}_{jt}$, $u^{\mathrm{su,pop}}_{jt}$, $u^{\mathrm{sd,pop}}_{jt}$, for $t \in T$, by maximizing the time t_1 to first startup or shutdown subject to Eq. (327), then fixing the commitment through time t_1, then maximizing the time t_2 to second startup or shutdown subject to Eq. (327), then fixing the commitment through time t_2, and so on.

For every producing or consuming device, the time-dependent real and reactive power bounds, the ramping limits, the constraints linking real and reactive power, the startup and shutdown trajectories, and the prior operating point real and reactive power values are mutually consistent under the prior operating point commitment solution:

$$\begin{aligned}&\text{There exist } p_{jt},\ p^{\mathrm{on}}_{jt},\ p^{\mathrm{su}}_{jt},\ p^{\mathrm{sd}}_{jt},\ q_{jt}, \text{ for } t \in T, \text{ satisfying}\\&\text{Eqs. (68) to (74), (80) to (89) and (98) to (128)}\\&\text{with reserve variables equal to 0 and commitment variables equal}\\&\qquad\qquad\text{to the POP commitment solution, for every } j \in J^{\mathrm{pr,cs}}\end{aligned} \tag{328}$$

7 Solver Requirements

The solver entry point should consist of a single code file, referred to as *Code*1, written in one of the supported languages. *Code*1 can in turn invoke other code files and libraries. *Code*1 should be invoked with a command taking certain arguments, which are listed in Table 9. The solver needs to read the problem input file, compute a solution, and write a solution file. The exact invocation syntax depends on the chosen language for *Code*1, as specified in the GO Competition languages webpage ([3]).

Table 9: Solver Command Arguments

Symbol	Description
*Code*1	*Code*1 is the filename of a script in one of the supported languages containing the entry point to the solver algorithm. The required filename depends on the chosen language (see [3]).
*InFile*1	*InFile*1 = "scenario_nnn.json" is the input file containing elementary problem data. Here "nnn" is three digits representing the scenario number between 0 and 999.
*OutFile*1	*OutFile*1 = "solution.json" is the output file containing solution data. *OutFile*1 should be written by the solver. Since the filename is specified here, *OutFile*1 is not provided as an argument to the solver command. It should be encoded in the solver.

Table 9 continued

Symbol	Description
TimeLimitInSeconds	*TimeLimitInSeconds* is the amount of wall-clock time in seconds before solver execution will be terminated. The solution file must be complete by this time. The value is either 600 for Division 1, 7200 for Division 2, or 14400 for Division 3 when executing $Code1$.
Division	*Division* = 1, 2, or 3. All Challenge 3 Divisions use Objective Function Scoring based on the market surplus maximization objective z^{ms}. The input files for a given scenario are different for each division.
NetworkModel	*NetworkModel* = "C3SvNxxxxxDn" is a 12 character string identifying the Network Model of the input files. These are the first twelve characters of the Dataset Network Model folder name. The purpose is to identify similar problem instances. Here "v" is a version number, "xxxxx" is five digits representing approximately the number of buses in the network, and "n" is the division number.
AllowSwitching	*AllowSwitching* = 0 or 1. If *AllowSwitching* = 0, then topology switching is not allowed, and all AC branch on/off status variables must be equal to the initial on/off status, i.e. $u^{\mathrm{on}}_{jt} = u^{\mathrm{on},0}_{j}$ for all $j \in J^{\mathrm{ac}}$, $t \in T$, and solutions violating this constraint will be deemed infeasible. If *AllowSwitching* = 1, then topology switching is allowed, and no further constraint is imposed. This argument is primarily for post-Challenge analysis to study the impact of switching. In order to ensure that the *AllowSwitching* argument is implemented by solvers and able to be used in post-Challenge analysis, we intend that there will be some runs contributing to Challenge 3 prize awards using *AllowSwitching* = 1, and there will be some using *AllowSwitching* = 0.

8 Solution Evaluation

Solution evaluation is performed by a Python code ([10]) that in turn relies on a data model code ([6]). Broadly, solution evaluation includes the following steps, requirements, and general organizing principles:

- Values of integer variables contained in the solution file should be written in the file as string tokens containing only decimal digit characters so that they can be directly parsed to integer values. In particular, there should be no decimal point.

- Values of continuous variables contained in the solution file should be written in the file as string tokens containing only decimal digit characters, possibly a decimal point, and may be in scientific notation, so that they can be parsed directly to double precision floating point values.

- Keys in the solution file should be exactly as specified in the format document ([11]) and should have exactly the UIDs of the variables specified in Section 5.4. There should be no repeated keys/UIDs and no missing keys/UIDs.

- The solution file should have no missing values and no extra values. Arrays should have the correct length. Every key should have a corresponding value.

- If the solution file is nonexistent or unreadable or incorrectly formatted or has incorrect contents as described above, then the solution is deemed infeasible.

- Values of variables contained in the solution file are read from the file and not modified thereafter.

- Values of variables not contained in the solution file are computed from variables that are already determined using the constraints and objective of the model as well as the optimality principle. That is, when a variables is not completely determined from the constraints, its value is assigned in such a way as to maximize the maximization objective. The objective variable z^{ms} is also computed in this way.

- When all of the variables entering a constraint have been determined, the constraint may be violated. If any constraint violation is greater than $\epsilon^{\mathrm{constr}}$, then the solution is deemed infeasible. This tolerance is intended to be large enough that errors of double precision floating point arithmetic in the solution evaluation process do not cause a solution to be deemed infeasible when it would be deemed feasible under exact arithmetic.

- Some constraints have slack variables written explicitly in the formulation and therefore cannot have any violation. These constraints are referred to as *soft constraints*. The slack variables on soft constraints appear in the objective with penalty coefficients. All other constraints are referred to as *hard constraints*. We may informally refer to the value of a slack variable for a soft constraint as the violation of that constraint, essentially viewing that constraint as a hard constraint. This informal point of view plays no role in solution evaluation.

- Evaluate constraints involving only discrete variables before those involving continuous variables.

- Base case and post-contingency connectivity constraints in Eqs. (59) and (60) are checked using the connected_components and bridges functions from the NetworkX ([4]) Python module ([1]).

- Constraints not included in the formulation but passed to solver through command arguments described in Section 7 (e.g. a requirement of no topology switching) are also checked, and if there are any violations, then the solution is infeasible.

- The post-contingency DC model is solved in each time period and each contingency for the post-contingency branch flows. This is by done factoring the DC bus admittance matrix and applying the Sherman-Morrison-Woodbury formula ([5]) for the inverse of a low-rank update of a matrix to the admittance changes resulting from topology switching in the branch on/off status variables and from contingency outages. This contingency evaluation technique for network changes of rank possibly greater than 1 is described in ([9]), and the ideas are similar to ([8], [7]).

- The post-contingency branch flow evaluation can be time consuming. Therefore, it is performed last, and it is skipped if the solution has already been determined to be infeasible.

- Then post-contingency branch flow limit violations and penalties are computed, and the post-contingency penalty objectives are formed and combined with the base case objective to form the total market surplus maximization objective z^{ms}.

- If the solution file is correctly formatted and no constraint violations greater than the tolerance ϵ^{constr} are found then the solution is deemed feasible.

- The solution evaluation procedure returns the feasibility status, the computed objective, and a set of output files containing any solution file errors that were encountered and a summary of constraint violations and important computed terms.

8.1 Solution Evaluation Output Files

The main script check_data.py produces the following outputs, among others.

- `summary.json`: contains a summary with various information about the problem file and the solution file being checked by check_data.py.

- `summary.csv`: a CSV-formatted version of summary.json where the column names are created by concatenating the corresponding keys in the JSON version.

The summary is a dictionary, and we give selected elements of the structure below, focusing on those that are most likely to be useful. New fields may be added from time to time, but generally fields will not be removed. The units are those of the formulation document.

- `summary`: The summary dictionary object, with the following fields.
  - `"problem_data_file"`: problem filename passed as an argument to the script
  - `"solution_data_file"`: solution filename passed as an argument to the script
  - `"git_info"`: information on the repository containing the script, e.g. commit date
  - `"problem"`: information about the problem, including formatting correctness and errors

- "solution": information about the solution, including formatting correctness and errors
- "evaluation": information about the solution evaluation, including feasibility of constraints, objective terms, and errors

- "problem": A field in summary that is again a dictionary, with the following fields
  - "general": General information about the problem
  - "violation costs": Soft constraint violation penalty coefficients
  - "num buses": Number of buses
  - "num ac lines": Number of AC lines
  - "num dc lines": Number of DC lines
  - "num transformers": Number of transformers
  - "num shunts": Number of shunts
  - "num simple dispatchable devices": Number of simple dispatchable devcieces
  - "num producing devices": Number of producing devices
  - "num consuming devices": Number of consuming devices
  - "num real power reserve zones": Number of real power reserve zones
  - "num reactive power reserve zones": Number of reactive power reserve zones
  - "num intervals": Number of time intervals
  - "num contingencies": Number of contingencies
  - "total duration": Total duration of the model time horizon
  - "interval durations": List of time interval durations
  - "error_diagnostics": Error messages if any errors were encountered in reading and checking the problem file
  - "pass": 1 if no errors encountered in reading and checking the problem file

- "violation costs": A field in summary["problem"] that is again a dictionary, with the following fields
  - "p_bus_vio_cost": Penalty coefficient on bus real power imbalance
  - "q_bus_vio_cost": Penalty coefficient on bus reactive power imbalance
  - "s_vio_cost": Penalty coefficient on branch overload
  - "e_vio_cost": Penalty coefficient on multi-interval energy constraints

- "solution": A field in summary that is again a dictionary, with the following fields
  - "error_diagnostics": Error messages if any errors were encountered in reading and checking the solution file

- "pass": 1 if no errors encountered in reading and checking the solution file

- "evaluation": A field in summary that is again a dictionary, with the following fields.
  - "viol_sd_t_u_on_max": Largest violation of upper bounds on u^{on}_{jt} based on Eqs. (48) and (52).
  - "viol_sd_t_u_on_min": Largest violation of lower bounds on u^{on}_{jt} based on Eqs. (48) and (51).
  - "sum_sd_t_su": Total of u^{su}_{jt} variables over all simple dispatchable (producing or consuming) devices.
  - "sum_sd_t_sd": Total of u^{sd}_{jt} variables over all simple dispatchable (producing or consuming) devices.
  - "viol_sd_t_d_up_min": Largest violation of minimum uptime constraints Eq. (57).
  - "viol_sd_t_d_dn_min": Largest violation of minimum downtime constraints Eq. (56).
  - "viol_sd_max_startup_constr": Largest violation of multi-interval maximum startups constraints Eq. (58).
  - "sum_sd_t_z_on": Total "on-cost" over all simple dispatchable (producing or consuming) devices.
  - "sum_sd_t_z_su": Total startup cost over all simple dispatchable (producing or consuming) devices.
  - "sum_sd_t_z_sd": Total shutdown cost over all simple dispatchable (producing or consuming) devices.
  - "sum_sd_t_z_sus": Total downtime-dependent startup cost adjustment over all simple dispatchable (producing or consuming) devices.
  - "viol_bus_t_v_max": Largest violation of maximum voltage constraints in Eq. (19).
  - "viol_bus_t_v_min": Largest violation of minimum voltage constraints in Eq. (19).
  - "viol_sh_t_u_st_max": Largest violation of maximum shunt position constraints in Eq. (137).
  - "viol_sh_t_u_st_min": Largest violation of minimum shunt position constraints in Eq. (137).
  - "viol_dcl_t_p_max": Largest violation of forward orientation DC line real power flow upper bound constraints based on Eqs. (152), (154) and (156)
  - "viol_dcl_t_p_min": Largest violation of forward orientation DC line real power flow lower bound constraints in Eqs. (152), (154) and (156)
  - "viol_dcl_t_q_fr_max": Largest violation of DC line from bus maximum reactive power flow constraints in Eq. (153).
  - "viol_dcl_t_q_fr_min": Largest violation of DC line from bus minimum reactive power flow constraints in Eq. (153).
  - "viol_dcl_t_q_to_max": Largest violation of DC line to bus maximum reactive power flow constraints in Eq. (155).

- "viol_dcl_t_q_to_min": Largest violation of DC line to bus minimum reactive power flow constraints in Eq. (155).
- "viol_xfr_t_tau_max": Largest violation of transformer maximum winding ratio constraints in Eqs. (144) and (147).
- "viol_xfr_t_tau_min": Largest violation of transformer minimum winding ratio constraints in Eqs. (144) and (147).
- "viol_xfr_t_phi_max": Largest violation of transformer maximum phase difference constraints in Eqs. (144) and (146).
- "viol_xfr_t_phi_min": Largest violation of transformer minimum phase difference constraints in Eqs. (144) and (146).
- "viol_acl_t_u_su_max": Largest violation of AC line upper bounds on u^{su}_{jt} based on $u^{\text{on},0}_{j}$ and *AllowSwitching*.
- "viol_acl_t_u_sd_max": Largest violation of AC line upper bounds on u^{sd}_{jt} based on $u^{\text{on},0}_{j}$ and *AllowSwitching*.
- "viol_xfr_t_u_su_max": Largest violation of transformer upper bounds on u^{su}_{jt} based on $u^{\text{on},0}_{j}$ and *AllowSwitching*.
- "viol_xfr_t_u_sd_max": Largest violation of transformer upper bounds on u^{sd}_{jt} based on $u^{\text{on},0}_{j}$ and *AllowSwitching*.
- "sum_acl_t_u_su": Total of u^{su}_{jt} (closing a circuit) variables over all AC lines.
- "sum_acl_t_u_sd": Total of u^{sd}_{jt} (opening a circuit) variables over all AC lines.
- "sum_xfr_t_u_su": Total of u^{su}_{jt} (closing a circuit) variables over all transformers.
- "sum_xfr_t_u_sd": Total of u^{sd}_{jt} (opening a circuit) variables over all transformers.
- "sum_acl_t_z_su": Total startup (closing a circuit) cost over all AC lines.
- "sum_acl_t_z_sd": Total shutdown (opening a circuit) cost over all AC lines.
- "sum_xfr_t_z_su": Total startup (closing a circuit) cost over all Transformers.
- "sum_xfr_t_z_sd": Total shutdown (opening a circuit) cost over all transformers.
- "sum_acl_t_z_s": Total flow limit overload cost over all AC lines.
- "viol_acl_t_s_max": Largest violation s^{+}_{jt} of AC line flow limit constraints, based on Eqs. (138), (140) and (141).
- "sum_xfr_t_z_s": Total flow limit overload cost over all transformers.
- "viol_xfr_t_s_max": Largest violation s^{+}_{jt} of transformer flow limit constraints, based on Eqs. (138), (140) and (141).
- "viol_bus_t_p_balance_max": Largest positive violation $\max(0, p_{it})$ of bus real power balance constraints, based on Eq. (17)
- "viol_bus_t_p_balance_min": Largest negative violation $\max(0, -p_{it})$ of bus real power balance constraints, based on Eq. (17)
- "sum_bus_t_z_p": Total real power imbalance cost over all buses.

– "viol_bus_t_q_balance_max": Largest positive violation $\max(0, q_{it})$ of bus reactive power balance constraints, based on Eq. (18)
– "viol_bus_t_q_balance_min": Largest negative violation $\max(0, -q_{it})$ of bus reactive power balance constraints, based on Eq. (18)
– "sum_bus_t_z_q": Total reactive power imbalance cost over all buses.
– "sum_pr_t_z_p": Total energy cost over all producing devices.
– "sum_cs_t_z_p": Total energy value over all consuming devices.
– "sum_sd_t_z_rgu": Total reserve procurement cost from all simple dispatchable devices for regulation up.
– "sum_sd_t_z_rgd": Total reserve procurement cost from all simple dispatchable devices for regulation up.
– "sum_sd_t_z_scr": Total reserve procurement cost from all simple dispatchable devices for regulation up.
– "sum_sd_t_z_nsc": Total reserve procurement cost from all simple dispatchable devices for regulation up.
– "sum_sd_t_z_rru_on": Total reserve procurement cost from all simple dispatchable devices for ramp up reserve when online.
– "sum_sd_t_z_rrd_on": Total reserve procurement cost from all simple dispatchable devices for ramp down reserve when online.
– "sum_sd_t_z_rru_off": Total reserve procurement cost from all simple dispatchable devices for ramp up reserve when offline.
– "sum_sd_t_z_rrd_off": Total reserve procurement cost from all simple dispatchable devices for ramp down reserve when offline.
– "sum_sd_t_z_qru": Total reserve procurement cost from all simple dispatchable devices for reactive power reserve up.
– "sum_sd_t_z_qrd": Total reserve procurement cost from all simple dispatchable devices for reactive power reserve down.
– "viol_prz_t_p_rgu_balance": Largest violation p^{rgu}_{nt} of regulation up balance constraints, based on Eqs. (20), (36) and (40)
– "viol_prz_t_p_rgd_balance": Largest violation p^{rgd}_{nt} of regulation down balance constraints, based on Eqs. (25), (37) and (41)
– "viol_prz_t_p_scr_balance": Largest violation p^{scr}_{nt} of synchronized reserve balance constraints, based on Eqs. (22), (38) and (42)
– "viol_prz_t_p_nsc_balance": Largest violation p^{nsc}_{nt} of non-synchronized reserve balance constraints, based on Eqs. (23), (39) and (43)
– "viol_prz_t_p_rru_balance": Largest violation p^{rru}_{nt} of ramping reserve up balance constraints, based on Eqs. (24) and (44)
– "viol_prz_t_p_rrd_balance": Largest violation p^{rrd}_{nt} of ramping reserve down balance constraints, based on Eqs. (25) and (45)

– "viol_qrz_t_q_qru_balance": Largest violation q^{qru}_{nt} of reactive power reserve up balance constraints, based on Eqs. (26) and (46)

– "viol_qrz_t_q_qrd_balance": Largest violation q^{qrd}_{nt} of reactive power reserve down balance constraints, based on Eqs. (27) and (47)

– "sum_prz_t_z_rgu": Total reserve shortfall penalty for regulation up.

– "sum_prz_t_z_rgd": Total reserve shortfall penalty for regulation down.

– "sum_prz_t_z_scr": Total reserve shortfall penalty for synchronized reserve.

– "sum_prz_t_z_nsc": Total reserve shortfall penalty for non-synchronized reserve.

– "sum_prz_t_z_rru": Total reserve shortfall penalty for ramping reserve up.

– "sum_prz_t_z_rrd": Total reserve shortfall penalty for ramping reserve down.

– "sum_qrz_t_z_qru": Total reserve shortfall penalty for reactive power reserve up.

– "sum_qrz_t_z_qrd": Total reserve shortfall penalty for reactive power reserve down.

– "viol_t_connected_base": Largest violation of base case connectedness constraints Eq. (59)

– "viol_t_connected_ctg": Largest violation of post-contingency connectedness constraints Eq. (60)

– "info_i_i_t_disconnected_base": Information on checking that the base case online in-service bus-branch network is connected in every time interval.

– "info_i_i_k_t_disconnected_ctg": Information on checking that the post-contingency online in-service bus-branch network is connected in every time interval and every contingency.

– "viol_pr_t_p_on_max": Largest violation of online producing device p-max constraints Eq. (109).

– "viol_cs_t_p_on_max": Largest violation of online consuming device p-max constraints Eq. (119).

– "viol_pr_t_p_off_max": Largest violation of offline producing device p-max constraints Eq. (111).

– "viol_cs_t_p_off_max": Largest violation of offline consuming device p-max constraints Eq. (121).

– "viol_pr_t_p_on_min": Largest violation of online producing device p-min constraints Eq. (110).

– "viol_cs_t_p_on_min": Largest violation of online consuming device p-min constraints Eq. (120).

– "viol_pr_t_p_off_min": Largest violation of offline producing device p-min constraints in Eq. (106).

– "viol_cs_t_p_off_min": Largest violation of offline consuming device p-min constraints in Eqs. (107) and (108).

- "viol_pr_t_q_max": Largest violation of producing device q-max constraints Eq. (112).
- "viol_pr_t_q_min": Largest violation of producing device q-min constraints Eq. (113).
- "viol_cs_t_q_max": Largest violation of consuming device q-max constraints Eq. (122).
- "viol_cs_t_q_min": Largest violation of consuming device q-min constraints Eq. (123).
- "viol_pr_t_q_p_max": Largest violation of producing device p-q-linking q-max constraints in Eqs. (114) and (116) to (118).
- "viol_pr_t_q_p_min": Largest violation of producing device p-q-linking q-min constraints in Eqs. (115) to (118).
- "viol_cs_t_q_p_max": Largest violation of consuming device p-q-linking q-max constraints in Eqs. (124) and (126) to (128).
- "viol_cs_t_q_p_min": Largest violation of consuming device p-q-linking q-min constraints in Eqs. (125) to (128).
- "viol_sd_t_p_ramp_dn_max": Largest violation of simple dispatchable (producing or consuming) device ramp down constraints in Eqs. (73) and (74).
- "viol_sd_t_p_ramp_up_max": Largest violation of simple dispatchable (producing or consuming) device ramp up constraints in Eqs. (71) and (72).
- "viol_sd_max_energy_constr": Largest violation of simple dispatchable (producing or consuming) device multi-interval maximum energy constraints in Eq. (75).
- "viol_sd_min_energy_constr": Largest violation of simple dispatchable (producing or consuming) device multi-interval minimum energy constraints in Eq. (76).
- "viol_sd_t_p_rgu_nonneg": Largest violation of simple dispatchable (producing or consuming) device regulation up nonnegativity constraints in Eq. (80).
- "viol_sd_t_p_rgd_nonneg": Largest violation of simple dispatchable (producing or consuming) device regulation down nonnegativity constraints in Eq. (81).
- "viol_sd_t_p_scr_nonneg": Largest violation of simple dispatchable (producing or consuming) device synchronized reserve nonnegativity constraints in Eq. (82).
- "viol_sd_t_p_nsc_nonneg": Largest violation of simple dispatchable (producing or consuming) device non-synchronized reserve nonnegativity constraints in Eq. (83).
- "viol_sd_t_p_rru_on_nonneg": Largest violation of simple dispatchable (producing or consuming) device online ramping reserge up nonnegativity constraints in Eq. (84).
- "viol_sd_t_p_rru_off_nonneg": Largest violation of simple dispatchable (producing or consuming) device offline ramping reserve up nonnegativity constraints in Eq. (85).
- "viol_sd_t_p_rrd_on_nonneg": Largest violation of simple dispatchable (producing or consuming) device online ramping reserve down nonnegativity constraints in Eq. (86).

– "viol_sd_t_p_rrd_off_nonneg": Largest violation of simple dispatchable (producing or consuming) device offline ramping reserve down nonnegativity constraints in Eq. (87).

– "viol_sd_t_q_qru_nonneg": Largest violation of simple dispatchable (producing or consuming) device reactive power reserve up nonnegativity constraints in Eq. (88).

– "viol_sd_t_q_qrd_nonneg": Largest violation of simple dispatchable (producing or consuming) device reactive power reserve down nonnegativity constraints in Eq. (89).

– "viol_sd_t_p_rgu_max": Largest violation of simple dispatchable (producing or consuming) device regulation up upper bound constraints in Eq. (98).

– "viol_sd_t_p_rgd_max": Largest violation of simple dispatchable (producing or consuming) device regulation down upper bound constraints in Eq. (99).

– "viol_sd_t_p_scr_max": Largest violation of simple dispatchable (producing or consuming) device synchronized reserve upper bound constraints in Eq. (100).

– "viol_sd_t_p_nsc_max": Largest violation of simple dispatchable (producing or consuming) device non-synchronized reserve upper bound constraints in Eq. (101).

– "viol_sd_t_p_rru_on_max": Largest violation of simple dispatchable (producing or consuming) device online ramping reserve up upper bound constraints in Eq. (102).

– "viol_sd_t_p_rrd_on_max": Largest violation of simple dispatchable (producing or consuming) device online ramping reserve down upper bound constraints in Eq. (104).

– "viol_sd_t_p_rru_off_max": Largest violation of simple dispatchable (producing or consuming) device offline ramping reserve up upper bound constraints in Eq. (103).

– "viol_sd_t_p_rrd_off_max": Largest violation of simple dispatchable (producing or consuming) device offline ramping reserve down upper bound constraints in Eq. (105).

– "viol_acl_acl_t_s_max_ctg": Largest violation s^+_{jtk} of AC line flow limit constraints under AC line outage contingencies, based on Eqs. (157), (159) and (160).

– "viol_xfr_acl_t_s_max_ctg": Largest violation of transformer flow limit constraints under AC line outage contingencies based on Eqs. (157), (159) and (160).

– "viol_acl_dcl_t_s_max_ctg": Largest violation of AC line flow limit constraints under DC line outage contingencies based on Eqs. (157), (159) and (160).

– "viol_xfr_dcl_t_s_max_ctg": Largest violation of transformer flow limit constraints under DC line outage contingencies based on Eqs. (157), (159) and (160).

– "viol_acl_xfr_t_s_max_ctg": Largest violation of AC line flow limit constraints under transformer outage contingencies based on Eqs. (157), (159) and (160).

- "viol_xfr_xfr_t_s_max_ctg": Largest violation of transformer flow limit constraints under transformer line outage contingencies based on Eqs. (157), (159) and (160).
- "z": Total penalized market surplus objective z^{ms}.
- "z_max_energy": Contribution to "z_penalty" from multi-interval maximum energy constraint violations.
- "z_min_energy": Contribution to "z_penalty" from multi-interval minimum energy constraint violations.
- "z_base": Contribution z^{base} to "z" $= z^{\text{ms}}$ from variables and constraints corresponding to the base (i.e. pre-contingency or no contingency) case.
- "z_value": Total value of energy consumption by consuming devices. Equal to the contribution to "z_base" $= z^{\text{base}}$ from consuming device energy value.
- "total_switches": Total of u^{su}_{jt} and u^{sd}_{jt} variables over AC branch devices, i.e. the number of topology switching actions.
- "z_cost": Total of all base case non-penalty costs including simple dispatchable (producing or consuming) device startup, shutdown, on, state-dependent startup adjustment, reserve procurement, AC line and transformer switching open and closed, and producing device energy costs. Equal to the negative of the contribution to "z_base" $= z^{\text{base}}$ from non-penalty costs.
- "z_penalty": Total penalties from bus real and reactive power imbalance, real and reactive reserve zone shortfall, AC branch overload, and multi-interval energy constraint violtions. Equal to the negative of contribution to "z_base" from soft constraint penalties.
- "z_k_worst_case": Contribution $z^{\text{ctg,min}}$ to "z" $= z^{\text{ms}}$ from the worst case contingency in each time interval.
- "z_k_average_case": Contribution $z^{\text{ctg,avg}}$ to "z" $= z^{\text{ms}}$ from the average case contingency in each time interval.
- "feas": 1 if the solution is feasible.
- "infeas": 1 if the solution is infeasible.
- "phys_feas": 1 if the solution is physically feasible, meaning that it is feasible according to the constraints of the formulation, and any violation of the soft real and reactive power balance constraints does not exceed ϵ^{constr}.
- "time_run": Run time in solution evaluation.
- "time_connectedness": Run time in evaluating connectedness constraints.
- "time_post_contingency": Run time in evaluating post-contingency constraints.
- "pass": 1 if no errors were encountered in the solution evaluation procedure.
- "error_diagnostics": Error messages encountered by the solution evaluation procedure.

- "infeas_diagnostics": Information about constraint violations resulting in a determination that the solution is infeasible.

- "infeas_diagnostics": A field of summary["evaluation"] that is again dictionary, containing constraint violations resulting in infeasibility. It is an empty dictionary if no such constraint violations are detected. If any such constraint violations are detected, then each one is a field in this dictionary with the following format.
  - "viol_*": The value corresponding to a "viol_*" entry of "evaluation" with "val" > 0.
- "viol_*": A field of summary["evaluation"]["infeas_diagnostics"] that is again a dictionary, containing information on the constraint violation of a particular type of constraints. The constraint type is indicated in the "*" text, as specified above, where possible referencing specific equations from the formulation. The fields of this dictionary are the following.
  - "val": The violation value of the most violated individual constraint of this type. Equal to $\max(0, f(x))$ for a constraint of the form $f(x) \leq 0$. Equal to None if the constraint array is empty.
  - "idx": The multi-dimensional index of the most violated individual constraint of this type. Equal to None if the constraint array is empty.
- "idx": A field of a "viol_*" dictionary that is again a dictionary with the following fields, representing a multi-dimensional index of an array of individual constraints indexed by multiple 1-dimensional index sets.
  - "0": The 0th entry in the multi-dimensional index
  - "1": The 1th entry in the multi-dimensional index
  - ...

9 Change Log

9.1 July 1, 2022

Extensive changes have been made to the formulation since the previous version (February 16, 2022). These changes are too numerous to list in detail here. In particular, the previous version had multi-mode devices, which were conceived of as a means for modeling combined cycle generators and storage devices. In the current version of the formulation, multi-mode devices have been removed. For trial event 1, multi-mode devices will not be a part of the formulation, and combined cycle generators and storage devices will not be modeled explicitly as such. Combined cycle generators, storage devices, and multi-mode devices may be added to the formulation later. Some, but not all, specific changes are listed here.

Added sections: Section 5 on data formats and construction, Section 6 on data properties, Section 7 on solver requirements, Section 8 on solution evaluation, and a change log in Section 9.

Reordered equations and added hierarchical structure and explanatory text in Section 4.

Changed network connectivity constraint in Eqs. (59) and (60), i.e requirement on solutions: In each time interval, in the base case and contingency, the bus-branch graph consisting of all buses and the AC in service online branches is connected.

Added network connectivity data property in Eqs. (232) and (233), i.e. requirement on data: In the base case and each contingency, the bus-branch graph consisting of all buses and the AC branches in service in the prior operating point is connected.

Removed all references to multi-mode devices and storage devices. Multi-mode devices and storage devices will not appear in the problem instances for Trial 1. These devices may be added to the formulation later.

Added constraints limiting reactive power reserve provision by devices based on reactive power dispatch and real power dispatch, for devices with linear constraints linking real and reactive power.

9.2 July 13, 2022

Some typos were corrected.

9.3 August 30, 2022

Limits on apparent power flow were incorrectly applied to DC lines. These limits and associated overload penalties have been removed. DC lines have limits on real power flow across the line and on reactive power at the from and to buses but not limits on apparent power.

9.4 December 31, 2022

Fixed some notation errors involving symbols missing superscripts or subscripts.

9.5 January 1, 2023

Added slack variables e_w^+ for the multi-interval maximum and minimum energy constraints in Section 4.6.3. These constraints are now soft constraints, and the slack variables appear in the objective with penalty coefficients c^{e}. To facilitate the inclusion of the penalties on these constraints in the objective, the variables and equations defining the objective in Section 4.1 have been revised. The substantive change in this revision is the addition to the objective of variables $z_j^{\mathrm{en,max}}$ and $z_j^{\mathrm{en,min}}$ for the penalties incurred for violating the multi-interval maximum and minimum energy constraints.

9.6 January 6, 2023

Sections for acknowledgments and an introduction (Section 1) have been added.

9.7 January 12, 2023

Added tolerance parameters with main letter ϵ and a minimum time unit d^{unit} that are used in checking elementary data, unambiguously constructing derived data, and evaluating solutions. In particular, note that ϵ^{constr} is the tolerance on violations of hard constraints used in evaluating the feasibility of solutions.

Added Section 5.1 on static data.

Modified inequalities in Section 5.3 with ϵ^{time} to ensure unambiguous data construction.

Added properties in Section 6 requiring that time values are integer multiples of the minimum time unit d^{unit}. These properties are needed to ensure that construction of derived sets of time intervals is unambiguous.

Added properties in Section 6 requiring that linear coefficients in constraints linking real and reactive power are not too close to 0.

Added properties in Section 6 requiring that real power values on startup and shutdown trajectories are not too close to 0. These properties ensure that the sets of time intervals associated with startup and shutdown trajectories are unambiguously defined.

Added properties in Section 6 requiring that β coefficients in linear constraints linking device real and reactive power are not too close to 0 or to each other. These properties are needed to preclude numerical errors in certain feasibility checks.

Added properties in Section 6 ensuring that the device energy cost and value functions cover the energy upper bounds and startup and shutdown trajectories.

Added properties in Section 6 ensuring that feasible startup and shutdown trajectories do not intersect.

Added properties in Section 6 ensuring that initial uptime and downtime, combined with minimum uptime and downtime, do not conflict with upper and lower bounds on on/off status variables.

Added properties in Section 6 ensuring that upper and lower bounds on commitment status, minimum uptime and downtime, initial uptime and downtime, and multi-interval maximum startups constraints are all mutually consistent. This property enables the definition of a fixed commitment solution, called the prior operating point (POP) solution, for each producing and consuming device.

Added properties in Section 6 ensuring that there exists a feasible dispatch solution for each producing or consuming device, given that the commitment variables are fixed to the POP commitment. This ensures that the problem as posed in this formulation document is feasible, possibly with nonzero values of the constraint slack variables included explicitly in the formulated constraints.

Rewrote Section 8 on solution evaluation. In particular, references to projection were removed, as we are not currently using that technique. Also, the role of the constraint violation tolerance ϵ^{constr} was specified.

9.8 January 13, 2023

Rewrote Section 7 on solver requirements. A table of solver arguments is now provided.

9.9 January 14, 2023

Added Eqs. (323) and (324), complementing Eq. (322). Together these properties ensure that the three terms p^{on}_{jt}, p^{su}_{jt}, and p^{sd}_{jt} have at most one nonzero.

9.10 January 22, 2023

Added a missing subscript in Section 4.6.3

Added missing $\forall$ statements in Eqs. (7) and (8).

Added Eq. (229) clarifying that the data format implies $J^{\text{pqmin}} = J^{\text{pqmax}}$.

Added some set disjointness properties near the beginning of Section 6.

Variables q^{qru} and q^{qrd} with additional superscripts and subscripts were in some cases incorrectly written as p^{qru} and p^{qrd}. These have been corrected. Symbols for quantities of reactive power reserve should use main letter q for reactive power, not p which is for real power.

9.11 January 26, 2023

Added an explanation that Eqs. (189) and (190) together constitute the definition of T^{out}_j for all $j \in J^{\text{pr,cs}}$, with each of them covering a subset of $J^{\text{pr,cs}}$, where these two subsets are disjoint, and their union is equal to $J^{\text{pr,cs}}$.

9.12 February 11, 2023

Fixed some typos.

9.13 February 26, 2023

Added Section 8.1 documenting the summary output files from the solution evaluation procedure.

9.14 May 15, 2023

Constraint relaxation penalties are now required to be strictly positive. Strict inequalities are used in Eqs. (281) to (292)

Added the field `"phys_feas"` to the field `"evaluation"` of `summary` in Section 8.1.

9.15 May 25, 2023

Added requirements on the data so that the start and end times of the multi-interval constraints are consistent with each other and with the end time of the model horizon. These are in Eqs. (307) to (312).

9.16 May 26, 2023

Added requirements on the data so that the time-indexed upper and lower bounds on dispatchable device reactive power do not prevent the reactive power reserve requirements from being satisfied with 0 constraint violation penalty. This is in Eq. (326)

9.17 January 22, 2024

Added/expanded sections - introduction (Section 1) and problem description (Section 2).

10 References

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